Modelling Bank Customers' Behaviour Using Data Warehouses and Incorporating Economic Indicators

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PhD – The University of Edinburgh – 2001

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Declaration

I declare that the contents of this thesis have been composed entirely by myself, that the work contained is my own, and that all contributions from others have been clearly indicated and have been due referenced.

Declaration of secrecy and confidentiality

I declare that I will at all times observe secrecy in respect of all the affairs of The Royal Bank of Scotland plc., and that I will not at any time disclose, reveal, cause the publication of or otherwise make use of such confidential information whether for my personal gain or otherwise.

Acknowledgements

This doctoral research would not have been possible and completed without the assistance of many people. First, I am indebted to my supervisor, Prof. Lyn Thomas, for his unfailing expertise and enthusiasm. I cannot recall a moment when he was other than intelligently helpful and strongly supportive. Further thanks are due to William Scherer of University of Virginia, U.S.A, for his constructive comments.

I appreciate the participation of the department of Risk and Lending Systems, The Royal Bank of Scotland plc. in this research. Many thanks to all members of staff there for making me feel welcome; special thanks to Lindsey Chalmers, Andrew Don, Alan Dunmur, Iain Knox, Chris Sykes and Stuart Young for their expertise. I had a pleasant and invaluable experience working with them.

Finally, I would like to thank my colleagues and other members at the Department of Business Studies, University of Edinburgh, especially those within the Credit Research Centre for their general support throughout my study.

Abstract

A credit risk monitoring model using Markov Chains was first prescribed by Cyert, Davidson and Thomson (1962) (the CDT model). It is used to monitor transition of a credit account from one performance state to another, as an alternative to scorecard methodologies. The propensity of such transition is called transition probability. Successive variants of the CDT model assumed a few outdated assumptions although proper tests had been available. Moreover no solutions were offered despite many had long suspected the dependency of transition probability on economic conditions.

In this empirical research, using real, substantial retail bank data, and adopting the Mover-Stayer notion (Frydman et al, 1985):

1. the unquestioned assumptions are proved invalid;
2. the true functional dependency of a transition probability time series on selected economic indicators is established;
3. the parameter associated to each explanatory variable is estimated using a non linear optimisation technique on the maximum logarithmic likelihood of a transition probability;
4. segments based on different transitional behaviour are identified for the given portfolio;
5. a pilot scorecard scheme is carried out to investigate membership to the segments identified in (4), given existing application and behavioural information.

1. Introduction

1.1 What is Credit and Behavioural Scoring?

"Credit Scoring" is a term used to describe the collection of techniques and processes that lenders use to assess the "creditworthiness" of an applicant. Primarily these deal with the decision of whether an applicant is suitable to receive credit. A product of such an exercise is a "scorecard" which gives a score to a particular attribute of the borrower. If the total score is above a pre-determined threshold (usually referred as "cut off" in the industry) then new credit is granted. It is a traditional classification problem where the outcome is usually binary; i.e. accept or reject, good or bad. As a result Credit Scoring techniques were originally based on statistical methods which construct classification rules such as discriminant and regression analysis, and classification trees. But other methodologies have also lent their hands. These include operational research methods like various optimisation techniques; and in the light of recent technological advances, artificial intelligence methods like neural networks and expert systems.

Behavioural Scoring is just a variant to Credit Scoring. Here applicants have become customers, and Behavioural Scoring deals with the decision of whether customers should be granted additional credit or facilities and whether a customer is likely to default. The approach and methods used to solve this problem are largely the same as Credit Scoring. On this occasion however instead of application details, past payment performance is considered, sometimes in conjunction with application details.

Lenders invariably face credit defaults. Minimising credit risk exposure has traditionally been the approach lenders use in choosing their portfolio. In today's cut throat market, (a consequence of recent deregulation, cost-saving mergers, hostile take-overs, conversions, and internet new entrants) accepting "good" customers who would not default is not necessarily a profitable option. Maximising profit is now the strategy. To do so lenders now have to accept reasonable credit risks by choosing a mix of good and "riskier" customers in their portfolio. Another variant of Credit Scoring, Profit Scoring, hence came about.

Thomas (2000), Hand and Henley (1997) and Rosenberg and Gleit (1994) provide a survey of the quantitative methods used in Credit and Behavioural Scoring. Capon (1982) offers a critical view on the subject. One could also refer to Lewis (1992), Mays (1998) or McNab and Wynn (2000) for the practical aspects of Credit and Behavioural Scoring.

1.2 The need for dynamics and economics in Credit and Behavioural Scoring

Credit and Behavioural Scoring suffer the intrinsic flaw of almost all forecasting models, in that past data is used to predict the future. This shortcoming is quite stark as a scorecard depicts only a static picture of the creditworthiness of the sample at the time of development. It is almost an universal practice to update a scorecard regularly (usually every 18 to 36 months) to counter the effect of economic changes to borrowers' attributes, i.e. "population drift". There still exists a time lag between the data collected and used in scorecard development and the actual implementation of the developed scorecard itself. By then the economic conditions could have been very different. Furthermore frequent updates still cannot fill the time gap in between and become very resource intensive. It is difficult to check updated scorecards are operating as expected. As a result incorporating economic knowledge into Credit and Behavioural Scoring presents a serious challenge to lenders who employ the methodologies. Other problematic aspects include biased samples (since the behaviour of those rejected is unknown), collinearity (attributes are linear function of each other), and in the case of Profit Scoring data warehousing since all elements of revenue and cost must be available in uniform format to analysts which may come from different parts of an organisation.

In Behavioural Scoring, an alternative approach to predicting default by a scorecard is to model the payment behaviour of customers in terms of his/her account "state" using Markov Chain models. These states usually describe the different stages of delinquency of the customers' accounts. The idea is to calculate the chance of payment or non-payment next period in each of these delinquency states for a homogeneous sample. This is a form of credit risk monitoring. All lenders need is the past and current delinquency states of the accounts in question. Thomas et al (to appear) describe how such model would solve the static problem of scorecards by transforming " *... snapshots of scoring into movie clips of consumer's behaviour ...* ". This approach is strengthened by the findings from Sullivan (1987) and Zandi (1998) who showed the dependency of defaults on economic conditions.

1.3 Conclusion

This thesis sets out to combine the Markov Chain approach and the economic dependency of credit defaults in building a sound borrowers' behavioural model. In Chapter 2 literature relating to the Markov Chain approach to credit behaviour will be reviewed and the context of this thesis will be set. In Chapter 3 the methodology to develop the samples for model development will be presented and particular economic dependency of delinquency for the samples will be investigated. The suitability of a Markov Chain model to the samples will be evaluated in Chapter 4, followed by the fitting of the resultant model. Test results in Chapter 4 will show that a more sophisticated segmentation of the samples and a higher order Markov Chain model are required. The search for an optimal segmentation and Markov Chain of suitable order is presented and discussed in Chapter 5 along with the results of model fitting to the new segments. In Chapter 6 a pilot scoring scheme will be tried to see whether one can predict the dynamics of borrowers' behaviour rather than just if they are good or bad. Finally conclusions will be drawn in Chapter 7 along with assessments and discussions on the methodologies used in this thesis.

2. Literature Survey

In this chapter the literature relating to the Markov Chain approach to credit behaviour will be reviewed and the context of this thesis will be set.

2.1 A General Description of a Markov Chain

Let $X = \{X(t), t = 0, 1, 2, \dots\}$ be a discrete time random process with a discrete state space S whose elements are $s_i, s_j, \dots$. X is a Markov Chain if the probability that $X(t+1)$ takes value $s_j \in S$ for any $t \geq 0$ is conditional only on the value of X(t) but does not depend on the values of X(t-1), X(t-2), ... Formally,

$$P\{X(t+1) = s_j \mid X(t) = s_i, X(t-1) = s_h, \dots, X(1) = s_g\} = P\{X(t+1) = s_j \mid X(t) = s_i\}$$

The one-time-step transition probabilities for a Markov Chain are:

$$\mathbf{P_{ij}(t) = Pr\ \{X(t+1) = s_j \mid X(t) = s_i\} \quad \forall\ i, j\ and\ t \geq 0} \qquad \textbf{Equation 2.1}$$

Equation 2.1 represents the probability that the process, when in state i, will next make a transition into state j. Probabilities are non-negative, and the process must make a transition into some state. This implies:

$P_{ij}(t) \geq 0\ \forall\ i, j$ and $t \geq 0$ and $\sum_j P_{ij}(t) = 1\ \ \forall\ i \geq 0$.

When transition probabilities are independent of t, they are called stationary, and the Markov Chain is referred to as time homogeneous. The process where X(t+1) depends only on X(t) is known as a First Order Markov Chain. The process where X(t+1) depends on X(t) and X(t-1) is known as a Second Order Markov Chain. This can be thought of as a Markov Chain by defining $Y(t) = \begin{bmatrix} X(t) \\ X(t-1) \end{bmatrix}$. Then saying

$P\{X(t+1) = s_j \mid X(t) = s_i, X(t-1) = s_h\} = P\{Y(t+1) = y_j \mid Y(t) = y_i\}$. Higher order Markov Chains can be defined in a similar way.

2.2 Markov Chain approach to credit payment behaviour

This section charts the development of the Markov Chain approach to credit payment behaviour in the literature.

The pioneering use of a Markov Chain as a representation of credit payment behaviour originated from the estimation of doubtful debts for accounting purposes at the end of a fiscal year in retail firms. The study was carried out by Cyert, Davidson and Thompson (1962) (CDT), the model presented is commonly referred as the CDT model. Two steps were involved. First, credit accounts were classified on a sample basis into age groups which reflect the stage of account delinquency. Second, the loss expectancy rate in each age category was estimated. Cyert and Trueblood (1957) had already formulated a framework for the classification process. This paper is the continuation of Cyert and Trueblood's work in seeking a scientific approach to perform the estimation of the loss expectancy rates. CDT defined the loss expectancy rates as judgmental estimates of the proportion of account balance in each age category liable to become uncollectable.

CDT acknowledged that "*... these loss expectancy rates are 'policy parameters' for they are not only based on past experience but are also functions of such things as the firm's expectations of economic conditions, conservation in financial policy, and other similar factors.*"[1]

CDT devised a method to estimate the loss expectancy rates and allowance to doubtful debts by developing a model which would describe the loss process of an

[1] Cyert et al (1962), p.287

account balance from being in a state of receivable to a state of being uncollectable or written off. They made use of the Markov Chain theory to answer three principle questions which are of interest to retail firm managers:

1) What fraction of the total receivables will become paid or bad debt?
2) What will be the eventual state distribution of new receivables by age category?
3) Sales vary cyclically. If new receivables are charged each period, what will be the distribution of receivables at the end of the period?

In order to answer these questions two assumptions were made. CDT "*...assumed that the matrix of transition probabilities is constant over time and independent of the initial age distribution of account balances; ... assumed that all accounts are the same size when 'Total Balance' method of ageing is used.*"[1]

CDT realised that the second assumption could be troublesome, which proved to be the source of dispute between two schools of thought in later works. It is the school of Total Balance Method and the school of Partial Balance Method. Under Total Balance Method, the account balance is put into an age category corresponding to the oldest charge. So a current purchase made by a customer with a three-month overdue balance would be classified into the three-month delinquent age category. Under Partial Balance Method, the account balance is apportioned among the age categories on the basis of the age of each of the charges.

CDT believed the second assumption was necessary, as it was not the individual dollars which move but rather all of the dollars in the account. This way a skewed account balance distribution would not distort the limiting probabilities in the transition matrices. CDT offered two remedies to counter the dispute. One is to stratify the account balances in to similar sizes before computing transition matrices. The second is to track the transitional behaviour of accounts rather than account balances. One overcame the argument by adopting the latter in this thesis.

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[1] Cyert et al (1962), p.290

The assumptions enabled CDT to calculate the steady state payment and amounts of bad debt in each period, and provide a model framework to accommodate situations where new sales were either constant or variable by period. CDT later was doubtful of the validity of their assumption of constant transition probabilities. They did not expect this assumption to be perfectly correct. They stated that transition probabilities of retail credit accounts are likely to be "*a function of changes in business activity ... it may be possible to predict the changes in transition probabilities, if any, by correlation with indices of local economic conditions.*"[1]

CDT concluded that transition probabilities and a Markov Chain as a description of accounts receivable behaviour provide a valuable insight into better ways of managing accounts receivables. CDT laid the foundation to later works by offering a means, which would give a complete picture of credit payment behaviour, that no other business indicators can.

Cyert and Thompson (1968) considered the scenario that a credit customer who never converts the credit into cash is not an asset to the firm. Their work reinforced the rationale that accounts receivables are the most important single asset on a retail firm's balance sheet. The use of retail credit in the US market was expanding rapidly then. Retail firms were caught in a dilemma between extending credit to customers, and the fear that poor credit quality will increase the firm's bad debt significantly. Cyert and Thompson (1968) criticised that no significant actions in credit management had taken place in the light of the increasing use and importance of retail and consumer credit in the US economy. Their first criticism is that the credit risks of new applicants were not assessed with respect to the portfolio or the different mix of existing customers. Their second criticism is that the potential net revenue from each applicant was never estimated. Their third criticism is there is always a probability of an account becoming uncollectable attached to each application for credit, but this probability was usually estimated subjectively by accountants. Cyert

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[1] Cyert et al (1962), p.300

and Thompson (1968) called for a greater control to be asserted by the credit management.

Cyert and Thompson (1968) presented a Credit Control Model, enabling retail firms to assert controls on granting credit by working out first the net expected and variance of revenue hence profit of an individual applicant. In this model, new applicants were classified into one of the pre-defined risk categories according to some "scoring function" (Credit Scoring). Each risk category was associated with its own transition matrix; the entries to these matrices were the transition probabilities of payment characteristic of that particular category. Then the coefficient of variance of the total expected (discounted) receipts was calculated. An applicant was granted credit from the lowest risk category to the highest until this coefficient fell below a predetermined value.

A set of assumptions similar to CDT was made. These were the Total Balance method, constant transition probabilities through time, and a Markov Chain process to describe the movement of dollars charged. The transitional processes were assumed to be probabilistic and independent. The transitional matrices described the payment behaviour of particular risk categories. Using a hypothetical sample and cost structure, Cyert and Thompson (1968) showed that as the initial variance of receipts increased the number of customers accepted in the higher-risk categories decreased. Cyert and Thompson (1968) suggested the initial variance could act as a parametric measure of the economic climate and business activities and that credit managers adjust to control the acceptance of applicants into the high-risk categories accordingly. Thus this method "*allows a firm to treat its credit customers as assets with differing risks, to select a portfolio of credit customers that meet an expected discounted profit-risk criterion for the whole portfolio ... rather than on the simple two category analysis of credit worthiness*"[2], i.e. accept or reject.

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[2] Cyert and Thompson (1968), p.45

Looking at credit management as a whole, little effort was spent on the credit collection side in the literature, whereas much has been done on the credit granting side. Mehta's (1972) effort concerned the former. He proposed in his study "*an expedient method based on Markov process for evaluating collection policy alternatives in a dynamic context*"[3]. Mehta (1972) correctly pointed out the weakness in the CDT model, namely that CDT had only considered the steady-state solution of a Markov Chain. The cash flow behaviour of retail credit accounts is unlikely to be static or to reach equilibrium instantaneously. As a result, results from the CDT model provide little meaning to describe the dynamic behaviour of credit payment.

Mehta (1972) considered both the transient and steady-state solutions of the Markov process. A steady-state solution represents the eventual amount of receipts a lender expects to receive. A transient solution reflects the amount of receipts collected so far at any time before the end of a predefined period. Any amount uncollected by then would be considered bad debt. Mehta (1972) presented a set of formulae which enables credit managers to determine the firm's optimal collection efforts at any time before writing off any accounts, under some predefined definition of cash flow. The model took into account the contribution margin on collecting receipts and incremental cost of bad debts, and an assumed objective of maximising the net present value of cash flows.

Mehta's (1972) work offered credit managers a means to draw up a comprehensive credit policy, integrating credit granting, extension and collection. In particular Mehta's (1972) model helps credit managers to decide the optimal length of time for recognition of bad debts in each age category of accounts, and to evaluate alternative collection policies, ultimately to achieve an effective allocation of resources.

Mehta's (1972) model requires the initial distribution of number of accounts in each age category, the transition probability matrix and the payment received during one

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[3] Metha (1972), p.38

credit period, to estimate both the transient and steady-state repayment cash flow after one credit period. From these Mehta (1972) calculated the net present value of receipts from all the accounts under alternative collection policies for a predetermined number of periods. Mehta's (1972) model might have operational relevance in helping credit managers to decide when to write customers off. However, Mehta's (1972) model puts strains on resources if the benefits of the optimal collection policy is not enough to outweigh the costs of estimating and updating the average and net receipts regularly.

Independently from Mehta (1972), Liebman's (1972) "Customer State Model" seeks an optimal credit control policy that minimises total credit costs by measuring the costs and returns from a choice of policies.

Liebman's (1972) construction of a "Customer State" involved classifying existing customers based on age of account, charge volumes and past payment experience. For each Customer State, transition probabilities and cost matrix were constructed and derived. Under alternative credit control policies, these were estimated and evaluated for comparison. A linear programming equivalent of the Customer State Model was also presented which offered parametric sensitivity analysis of the optimal policy to changes in costs or in transition probabilities. This information will be useful to management as to whether to change or modify the current credit policy.

Liebman (1972) stated his effort did not concern the credit granting side of credit management, but suggested two areas for future research which could integrate the granting decision into his model, making a credit policy truly comprehensive. One is the explicit consideration of new account acceptance decision in the model. Second is the extension of the formulation to include marketing policies within the model's framework.

Cyert and Thompson (1968) using a scoring function to classify existing customers into different risk categories already tackled the first aspect. The second concerns the trade-offs between potential total credit costs and potential contributions from receivables, which had been dealt with, by Mehta (1972).

Corcoran (1978) was concerned with accounting control on account receivables and cash budgetary forecast. Corcoran (1978) assumed a dynamic transition matrix and Partial Balance method which was the usual practice at the time, but he found no basis for CDT's comment that it is not necessary to group accounts into the same size if the Partial Balance method is used. Corcoran (1978) argued "*splitting a balance into portions does nothing to alleviate the dominance of large accounts, but it remains a useful idea to stratify accounts prior to preparing transition matrices*"[4].

Corcoran (1978) applied exponential smoothing to transition matrices to improve the Markovian estimates of the month to month cash flow and changing customer payment behaviour. Modification to account for seasonal and trend adjustments was offered utilising Winters' (1960) equations. Corcoran's (1978) initial data showed there was noticeable variation in both the monthly balance and their ageing. He argued that exponential smoothing could correct this instability and offer a more representative transition matrix. The main attraction of Corcoran's (1978) exponential smoothed transition matrices is the use of a smoothing constant. This means recent payment behaviour is emphasised. Trend and seasonal corrections provide means for adjusting predictions from previous years.

Corcoran (1978) too called for a search for an explicit relationship between payment behaviour and the economic climate as well as company policies: "*For changes in paying behaviour, one looks for changes in customer's business cycles, tightness of money in the economy ... and where dealings are directly with individual customers – such causes as Christmas, summer vacation bills ...*"[4].

CDT and Corcoran's (1978) respective assumptions are steady-state behaviour and Total Balance method, and dynamic transient behaviour and Partial Balance method

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[4] Corcoran (1978), p.734

[4] Corcoran (1978), p.738

respectively. Each makes sense for the problem considered by the respective authors. It is because, in accounting terms, what the respective authors were seeking to predict are two different items in an accounting statement. CDT's interest was an item on the balance sheet, i.e. provision for doubtful debts, which is a snap shot of the financial position of a firm at a particular date. Corcoran's (1978) interest on the other hand was the amount of cash flow in a period of time.

In yet another attempt to tackle the weakness in CDT's Total Balance assumption, Van Kuelen et al (1981) criticised CDT for using the Total Balance method to estimate the 'real' age distribution of accounts receivables, in doing so CDT failed to distinguish the "total balance age" and the "real age" of the dollars in an account. Van Kuelen et al (1981) illustrated with a set of fictitious account balances that CDT's Total Balance method underestimated the actual amounts of dollars paid. They stated that the "ageing flaw" was a result of not recognising partial payment of the balance. Their solution aimed to strike a balance between the Total and Partial Balance method by treating " *...partial payments separately in the paid category and age the remainder of the former balance according to the age of the oldest invoice*"[5]. They tested their new approach on both a fictitious data set and CDT's original figures, accuracy in prediction was improved in both cases.

2.3 From monitoring cash movement to monitoring transactional behaviour to Mover-Stayer model

Kallberg and Saunders' (1983) interest was in payment behaviour as a whole. They were not concerned with the amount paid at all, which was a change from the usual approach in the past. The probability of an account moving between delinquency states was observed and studied, instead of the probability of a dollar moving between delinquency states. This was actually a realisation of CDT's criticism on their own model. Their approach stemmed from the differentiation between "micro

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[5] Van Kuelen et al (1981), p.110

data" (data on individual transitions) and "macro data" (data aggregated into each performance state).

Three variations of transition matrices under three different state-space definitions were presented: the original CDT definitions, an extension to the CDT definitions by expanding the current state conditioned on the opening balance of the account, and a completely new definition based on the delinquency of payment only. "Global" transition matrices were produced to investigate the steady-state or average payment behaviour. "Periodic" transition matrices were produced to study the dynamics of payment behaviour. Charts of transition probability series against time were first presented in this paper which depicted the evolution of transition probabilities over time.

Attempts were made to correlate the changes in transition probabilities observed to the changes in the economy in a non-scientific, commentary fashion. Kallberg and Saunders (1983) acknowledged a formal model would provide direct information on the causal factors of payment behaviour. But it was beyond the scope of their study at the time and they were restricted by the limiting size of their data sample: "*$P_{01(t)}=f(r_t, U_{t-1}Z_t)$ ... regressing the estimated values of $P_{01(t)}$ on some vector of interest rates (r), unemployment levels (U), and other macro and/or seasonal variables felt to be appropriate (Z)*"[6]. An explicit relationship between economic indicators and transition probability series incorporated into a transition matrix facilitates an accurate forecast of the individual entries in the matrix.

Their intended analysis of the chosen transition probabilities showed an overall trend with cyclic movements in between. Kallberg and Saunders (1983) argued the size-effect tends to over aggregate information on account movements. They pointed out specifically that an account with a small opening balance might be more likely to be paid off in the next period than one with a large opening balance.

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[6] Kallberg and Saunders (1983), p.10

This led Kallberg and Saunders (1983) to investigate whether a size-effect does exist. They subdivided the current state according to the size of the opening balance. The resultant Global transition matrix showed a positive relationship between the transition probabilities of moving from current state to being one month overdue and the size of opening balance. The charts of transition probabilities for the transitions from the two current states with the smallest opening balance to the fully paid state showed no obvious trend or seasonality.

In the final part of their investigation, Kallberg and Saunders (1983) devised a new state-space definition to investigate directly the payment behaviour. New states were defined such that several types of distinctive payment behaviour could be easily identified: inactive, revolver, those paying off less than the minimum required amount and no payment at all. From the Global transition matrix, Kallberg and Saunders (1983) showed one can estimate the eventual distribution of the accounts falling into one of the above behavioural categories. Under the new state-space definition, the charts of transition probabilities against time showed noticeable changes in the initial stage of the study period. Kallberg and Saunders (1983) postulated that events in the US economy at the time of study might be the cause of these changes.

This paper is a milestone in taking the Markov Chain approach to credit payment behaviour since CDT. It has led the way to shift the focus of attention from cash forecasting and accounts receivable planing and control, to the overall behaviour of individual type/group of credit customers. In particular, they called for formal tests to be carried out on the suitability of the Markov Chain model as a representation of credit payment behaviour. Kallberg then collaborated with Frydman and Kao in producing a paper, which provided another significant change in the use of Markov Chains in modelling payment behaviour.

A stationary Markov Chain model based on the CDT framework has become the benchmark in the context of modelling credit payment behaviour. The assumptions associated with it were readily accepted without rigorous assessments for validity.

Frydman et al (1985) listed two reasons as the rationale for their study: "*changes in usury ceilings, interest rates, or in purchase patterns may lead to non-stationary payment behaviour, ... the population of credit account is likely to be heterogeneous with respect to payment behaviour*"[7]. Frydman et al (1985) hence proposed two alternatives: a non-stationary Markov Chain and a Mover-Stayer model, to be tested against primarily the benchmark, a stationary Markov Chain model, and each other for suitability in describing credit payment behaviour. The tests used are the likelihood ratio test and residual matrices formulated by Anderson and Goodman (1957), and Frydman (1984), with a three states state-space definition on some retail revolving credit accounts data.

The "Mover-Stayer" notion originated from a labour mobility study by Blumen et al (1962). The assumption made in the Mover-Stayer model was that the population is comprised of two groups of accounts sub-population:

"*Stayers – individuals who never leave their initial states;*
Movers – individuals who make transitions according to a stationary Markov Chain"[7]

In the context of credit payment behaviour, Stayers are those customers who prefer to pay up fully each month; Movers are those customers who make partial payment which may or may not meet the minimum required amount requested by lenders.

The results from the likelihood ratio test, given the data, showed the Mover-Stayer model was superior to the stationary Markov Chain. The results from the analysis of residual matrices showed stationary and non-stationary Markov Chain models were comparable to each other, based on the criterion of percentage error computed from the differences between predicted and observed transition probabilities. Frydman et al (1985) also formulated likelihood ratio tests to examine the goodness of fit of each model to the data used. However they found that it was not possible given the data

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[7] Frydman et al (1985), p.1204

[7] Frydman et al (1985), p.1204

they had. They explained that the limited applicability of such tests was due to the number of possible transition histories is usually large relative to the sample size. Some histories would have no or few entries in the frequency tables as a result. A small state-space was used (three states) in this study. Frydman et al (1985) did not consider the possibility of "structural zeroes" which are zero entries due to theoretically impossible transitions as mentioned in Weiss et al (1982), who provided corrective formulae to calculate the degrees of freedom in the data presented in a frequency table with zero entries.

Frydman et al (1985) found that, given the data, state-space definition and context, "*incorporating heterogeneity into the model is more important than incorporating non-stationarity*"[7], and that a Mover-Stayer model served a better representation of credit payment behaviour. They also acknowledged while a Mover-Stayer model incorporates a simple and meaningful population heterogeneity in the given context, there may be other forms of heterogeneity in other scenarios.

From a marketing perspective of the banking industry, Schniederjans and Loch (1994) outlined the fundamental changes taking place in and new challenges facing the US banking industry. The main determinant was deregulation which changed the definition and operation boundary to the traditional banks. The UK followed these changes in the 1990s. The net effect was increased competition which drove down profit margins. At the same time customers became more informed and educated in the new information age. They pointed out " ... *increased customer demand for more varied and elaborate services have placed bankers in the new position of having to develop a marketing culture ... to aid in the planning and directing of resources to those specific areas that present the best opportunities for growth in profits*"[8].

Schniderjans and Loch (1994) suggested banks should adopt a marketing strategy via segmentation by product service to find out and understand the customer base the

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[7] Frydman et al (1985), p.1203

[8] Schniederjans and Loch (1994), p.281

bank wishes to serve, and accommodate all of the customer's needs. The aim of this study was to explore the use of a Markov projection model in bank service usage as a means to generate evidence of needs and profile of existing customers.

Schniderjans and Loch (1994) using real data from a medium sized US bank found " *the long term dynamic nature of the current real world banking industry and the use of stationary Markov analysis is inconsistent*"[8]. They went on to test the non-stationarity in their data using the goodness of fit test adopted by Frydman et al (1985). The results checked favourably. They cast doubts however on the long term projection capabilities of a non-stationary model, which utilised the current distribution of customers using different types of service, the current and the last period transition matrix. It is because all of these need to be updated regularly. In the process of analysing the transition matrices, Schniedrejans and Loch (1994) too found the difficulty of small and zero entries, which were the result of the small sample, size. On "lumping" the states together into broader categories, they found the results produced were over generalised, and some were questionable in particular regarding the time a customer using the same service again.

The contribution of Schniederjans and Loch (1994) and Frydman et al (1985) was to provide evidence that refuted the use of a simple Markov Chain model in real world applications.

2.4 Data Mining

This section will briefly introduce different views from authors of different disciplines on Data Mining and its connection with this thesis. The connection lies in the fact that today's retail banks and firms alike which gather huge amounts of data about their customers realise that it will only be beneficial to do so if they can find a way to extract information from it. This information will tell, in this case, lenders about the borrowing and payment behaviour of their customers. When using the

[8] Schniederjans and Loch (1994), p284

Markov Chain approach (section 2.2), future customer behaviour can be predicted given their past behaviour.

Reporting on the growth and the current trend in the data mining industry, Mullich (1997) explained that the rising popularity of data mining was partly due to the technological progresses that allow faster, more efficient analysis of databases. The other was due to the thirst of businessmen and scientists alike to analyse and interpret digitised data for their specific purposes quickly and much less expensively. Discovering meaningful and useful information from operational data is the main aim of a data mining exercise. It has been used to predict credit default risks, to find geological earthquake patterns, to forecast inventory demands, or even used by observatories to identify stars and galaxies.

A data mining or knowledge discovery exercise would produce something worthless if there is not a business involved, which demands and consumes the new found knowledge. Therefore Smith (1999) described data mining as a business process. A process which is "*all about helping business to extract meaningful relationships and information from huge databases in order to gain competitive advantage. It is a methodology for discovering hidden information, and a completely holistic approach to better understanding the meaning of the data collected by a business*"[9].

In the last decade or so, businesses have relied more and more on computerised transaction systems to conduct their daily operations. Accelerating advances in computer storage and processing power fuelled the growth in data warehousing. To those businesses who collect masses of data, efficient and automated data mining is a logical step forward to unearth new, non-apparent relationships concealed among vast bodies of data, in order to achieve a competitive edge over competitors in today's cut-throat business environment. The idea of extracting information from data is not new. But the phenomenal interests placed in and the rise of contemporary data mining techniques, Smith (1999) explained, are a result of these techniques

[9] Smith (1999), p94

being not as stringent on rigid, sometimes limiting, assumptions as required by traditional statistical techniques. Furthermore, there have been numerous examples of successful application where these techniques outperformed the old ones. However there have also been examples where this is not the case or too much was expected of data mining.

Smith (1999) acknowledged that data mining techniques are grounded on statistics and mathematics. She claimed that it was dangerous to treat data mining techniques as a "black-box", which is something vendors tend to market their products as. To this end Smith (1999) shared the same view as Mullich (1997). In order to get the best out of data mining techniques, it was important for the users to understand the architecture of the model, to ponder what effects of changing the model parameters will have on the results. Smith (1999) stressed that data mining should be carried out in consultation of business knowledge and goals. It is because there is no point in doing the exercise if the new found information is irrelevant or not actionable.

Smith (1999) called the actual statistical tool(s) performed in the analysis or mining of data to extract information related to the goal in a data mining project, a "Knowledge Discovery Algorithm" (KDA). The application of a KDA could involve one or more traditional or contemporary statistical techniques, such as discriminant analysis, regression, neural networks and classification tress, etc.. KDA can be "Directed" where there is a predefined set goal which will direct the knowledge discovery process. Classification and prediction problems are examples where Directed KDA like regression, decision trees can be utilised. KDA can also be "Undirected" where the knowledge discovery process proceeds without any directions implied or given by analysts, or there are some unknown underlying patterns, concealed features of the data users wish to uncover. Clustering problems are examples where Undirected KDA like neural networks, and k-means algorithm can be utilised.

Hand (1999) provided a critical survey on the subject. He was wary of "*whether there is anything new to the area of data mining. Are its practitioners offering* *something new, tackling problems different from those tackled by statisticians?"*[10] He believed there are serious theoretical issues emerging in data mining problems which could benefit from a statistical perspective and understanding. Under the adopted theme of "Greater Statistics" which means "learning from data", Hand (1999) defined data mining as a process of discovering unexpected, valuable, or interesting structures or patterns in large data sets.

Hand (1999) identified the large size of data sets nowadays gathered by scientists, medical practitioners and retail firms alike as being the propelling force behind the hyped popularity and interests in data mining. The large size of datasets was also the root cause to the different approaches to data analysis taken by statistics and data mining. Hand (1999) was particular in clarifying a data set is "large" if it is entirely not possible to analyse it by hand, and where computers and data analytic software are an essential tool for analysis.

Hand (1999) distinguished the similarities and differences between the two disciplines. Data mining is focused on data exploration and description. Modern statistics however is focused on "mechanistic modelling" and inference. In data mining problems it is not unusual for analysts to be presented with a whole population of cases, so sampling would become irrelevant. The nature of inference would dramatically have changed as one now is not hypothesising what underlies the population, but rather trying to extrapolate the future. By nature, applications of formal statistics tend towards abstracts. Many researchers invent a tool and then seek a data set on which to illustrate. The use of data mining on the other hand is more application specific and tangible.

Hand (1999) believed as a result of data mining's parentage which includes computational sciences, far greater emphasis is placed in the role of the algorithm on a much faster time scale. Hand (1999) criticised data mining practitioners who rushed to design and create new algorithms with little appraisal or comparison with

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[10] Hand (1999), p.21

alternative methods. Hand (1999) felt that this was to some extent a result of the advances in computer technologies in that it is not too difficult to invent and programme, because they were driven by fierce competition in a commercial market place and the notion of "some solution is better than none".

In conclusion Hand (1999) warned data mining practitioners on developing new algorithms "*in the absence of a unifying theoretical underpinning analogous to that underlying statistical modelling*"[10]. And this is where he believed the theories of "Lesser Statistics" (specific statistical methodologies) and the expertise of conventional statisticians could contribute to the development of the relatively young discipline.

2.5 Conclusion

The Markov Chain approach to describe credit behaviour has traditionally, and primarily, been involved in debt provision. It still is. The emphasis has shifted from tracking money movement to monitoring behavioural movement. Financial services providers are trying to minimise the risks of being exposed to defaults and bad debt. In the current market place, they are also putting efforts into retaining loyal, good customers from leaving to rival competitors. Loyal, good customers are not necessarily profitable customers; lenders would have lost market share and businesses to rivals otherwise. To maximise profit, lenders need to accommodate a portfolio of different mix of customers with wide range of credit risks, and to tailor different products and strategies to serve different type of customers. This has some bearing to Customer Relationship Management (CRM), but this is out of the scope of this thesis.

As Thomas et al (to appear) pointed out, a state space that describes all the different possible situations that a customer can experience is most crucial for a Markov Chain

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[10] Hand (1999), p.27

model to provide believable results. Dealing with payment behaviour as a whole as assumed in this thesis should overcome the ageing argument.

The argument between those who believe in a steady state and a transient solution is only a practical one, dependent on the purpose of the study. A stationary Markov Chain model had been readily accepted without question in the past. But it was Cyert et al (1962) themselves who first recognised that transition probabilities are probably a function of business and economic activities. No formal tests for the model's suitability had been performed until Frydman et al (1985), some twenty years after CDT's publication. Thus the second crucial assessment Thomas et al (to appear) stressed is to ascertain credit behaviour does exhibit Markovity, while stationary or non stationary.

Authors in previous works had hoped for an explicit, direct relationship between transition probabilities and economic conditions or business policies. They were aware that transition probabilities are unlikely to be constant through time. The long term projection capabilities of a non-stationary Markov Chain model depends on how far ahead transition probabilities can be accurately forecasted. Integrating credit granting/extension and collection decisions into a credit behaviour model is also an achievement long desired.

Thanks to the participation of a major UK retail bank, unlike some of the previous works where small (sometimes fictitious) data sets had caused difficulty, a large account performance data set over a substantial time period (close to a business cycle) was available to this thesis for analysis.

Data Mining has been described as a process. Data mining cannot be performed in isolation without the support of business knowledge. Data mining cannot escape its founding theories from statistics and mathematics although a computer is a necessary medium. The thirst for relevant, actionable information in competitive markets has been identified as the cause to the rapid development in data mining. Knowledge discovery leads to subjects further afield like Customer Relationship Management (CRM), and Customer Value Management (CVM).

This thesis is an extension to previous works regarding the Markov Chain approach to describe credit payment behaviour. From here onwards, "credit (payment) behaviour" means the "dynamic movements between delinquency states" of credit customers.

This thesis sets out in the following chapters to accomplish the shortcomings in previous works. To do this:

- formal tests will be performed to refute the constant transition probabilities assumption;
- Mover-Stayer notion is assumed; Movers' behaviour will be tested whether they exhibit Markov behaviour;
- a non stationary Markov Chain model will be constructed for the Movers;
- an explicit relationship between transition probabilities and some economic indicators will be constructed;
- scorecards will be built to predict in advance the likelihood of a customer becoming a Stayer/Mover in the coming future;
- Conclusions will be drawn from the empirical results and discussion will be provided.

3. Methodology

3.1 Introduction

In Chapter 2 the background to the Markov Chain approach to represent credit behaviour was explained. It described its connection to credit/behavioural scoring and data mining. The chapter has put this thesis into context. Its conclusions outlined the agenda of this thesis. In this chapter, the steps taken to achieve the objectives set out in Chapter 2 will be detailed and discussed.

The Royal Bank of Scotland plc (RBS) was the data source of this thesis. The RBS is the retail banking arm of the parent – The Royal Bank of Scotland Group plc, which is one of the major financial services groups in the UK. The RBS offers consumers a wide range of credit products. These include overdrafts, mortgages, credit cards, charge cards, and personal loans. Lenders like the RBS are exposed to credit risk. Lenders make loans on nothing more than a "promise" to repay (Rose, 1999). Borrowers' failure to make some or all of their promised interest and principal payments results in an erosion to a bank's capital. The capital of a bank and the risks it is exposed to are closely related to each other. Moreover, deposits generate cash reserves, and surplus reserves are the major ingredient for bank loans and other investments. Therefore deposits are a distinctive item on a bank's balance sheet.

Consequently, this thesis concentrated efforts on the Money Transmission Accounts (MTA) of the RBS. MTA are a typical transaction deposits product, or better known as personal current accounts. The services namely include making payments or deposits on behalf of the customers. This requires the bank to honour any withdrawals or deposits made by the customer in person or by a designated third party. These accounts often come with overdraft and charge card facilities, and as a result they can be seen as a borrowing instrument. Repayments to other loan products, such as mortgages, motor finance, credit cards, etc. are likely to come out of a customer's current account. The performance of a customer's current account is a good indicator which reveals difficulties in meeting payment requirements in others.

Lenders have traditionally used credit and behavioural scoring to monitor the performance of their portfolio. However, updating scorecards frequently is not an economic solution to counter population drift caused by the overall economic climate. Therefore as a major objective, this thesis set out to provide an alternative and a more dynamic approach.

3.2 Creating the target data sets

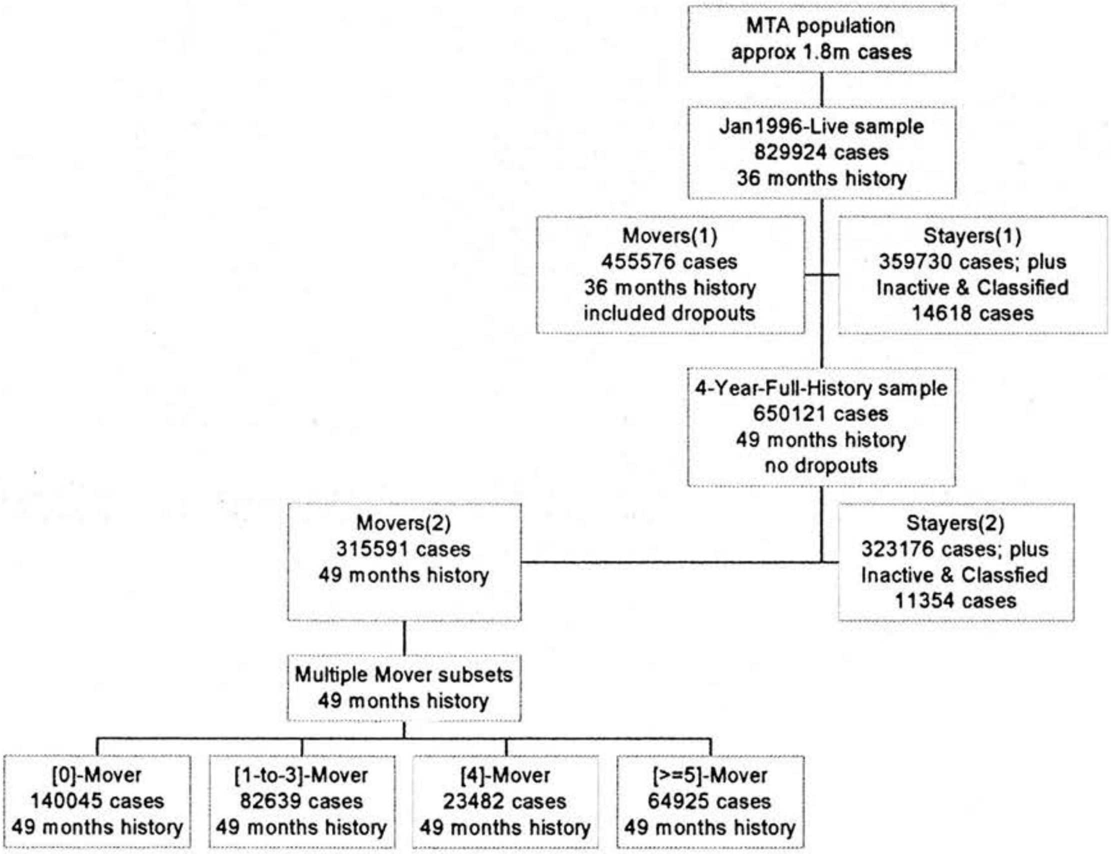


Figure 3.1 A flow chart to illustrate how target data sets were created

The target data sets created were sets of longitudinal monthly PROBE states (PROBE states outlined in Appendix 9.1, but for confidentiality reasons they cannot be precisely defined) which described the delinquency states of an account over a time horizon. They originated from the MTA population which was live as of the month end January 1996. The target data sets described the account performance history of an account as from January 1996. The steps taken to create the final target data sets for the analysis follow in the next few sections. Figure 3.1 depicts how they are related. Characteristics of the customers in the target data sets were defined as follows:

STAYER were those accounts which were alive, active and stayed current (PROBE state = 1) throughout the time span of the study period.

INACTIVE were those accounts which were alive and stayed inactive throughout the time span of the study period.

CLASSIFIED were those accounts which were alive and stayed in the recovery state throughout the time span of the study period.

MOVER were those accounts which were alive, active and not STAYERS, not INACTIVE and not CLASSIFIED throughout the time span of the study period.

DROPOUTS were those accounts closed or written off during the time span of the study period.

Multiple Mover Subsets are daughter subsets of the parent Mover(2) sample. They will be discussed later in section 3.2.5 and Chapter 5.

Inactive and Classified accounts were not involved in any of the analysis in the following chapters. It is because they were either inactive or in recovery state for a very long period of time. A full dissection of the Jan1996-Live and 4-Year-Full-History sample are offered in Figure 3.7.

3.2.1 Jan1996-Live sample

A quota sample of the MTA population defined by computer memory space limits was extracted from the RBS database, "Jan1996-Live" sample. The time horizon for this particular sample was the period between January 1996 to December 1998. Each RBS account has an unique internal identification number attached to it which is a sequence of integers. This identification number is unique to accounts but not to customers. It is possible for a customer to hold more than one MTA account, but this possibility is considered very small. When an account is closed, the associated identification number will be discontinued for a period of time before being reissued. The sampling was done by choosing random account numbers. There did not seem to be any observation bias in the accounts. No new accounts or closed accounts were allowed to (re-) enter the Jan1996-Live and its daughter samples. So in this thesis one was studying the credit behaviour of a cohort of the MTA population as of January 1996.

The first sample constructed was used in a training exercise designed to allow the Author to get used to the operation and environment of the complex RBS database. The only restriction was that customers should not have been promoted from a Cashline to a Highline cash card in the period of January to July 1996. This exercise then compared the credit behaviour of those accounts which were promoted from Cashline to Highline in the January to July 1996 period with that of the Jan1996-Live sample. The results of this study will not be presented here but the sample of customers obtained was used for the preliminary investigation in the thesis.

The Jan1996-Live sample consisted of different types of MTA products, joint or single accounts, ages of accounts, facilities and overdraft limits, etc.. including those who closed their accounts or were written off by the RBS (Dropouts) during the period of the time horizon. The sample was an assortment of the MTA population portfolio as at the month end January 1996. The resultant empirical model is therefore a fair representation of credit behaviour of the month end January 1996 MTA portfolio.

PROBE state	Frequency	Percentage (%)
0	63893	7.7
1	644948	77.7
2	76114	9.2
3	6765	0.8
4	730	0.1
5	14222	1.7
6	5220	0.6
7	1537	0.2
8	5302	0.6
9	5690	0.7
10	4890	0.6
11	517	0.1
12	96	0.01
Total	829924	100

Figure 3.2 Distribution of accounts among performance states for the Jan1996-Live sample as of the month end of January 1996

Figure 3.2 depicts the distribution of accounts among performance states for the Jan1996-Live sample as of the month end of January 1996, i.e. the starting point of the study period. It can be seen that a majority of accounts started off being current (PROBE state = 1). Those started off closed (PROBE states 10, 11, 12; less than 1%) were not reinstated during the period February 1996 to December 1998 (the study and test period for Movers(1) derived from this sample, see section 3.4), i.e. the associated identity numbers were not reissued.

3.2.2 Movers(1) and Stayers(1)

The Mover-Stayer population heterogeneity was implemented on the Jan1996-Live sample by splitting the sample into two distinctive groups: Movers(1) and Stayers(1) (section 3.7). Stayers are those customers who stay current (PROBE state = 1) constantly all the time. Movers are those customers who make transitions from time to time. The definitive definitions are offered in section 3.7.

Analysis as described in Chapter 4 was performed on the Movers(1) sample. Results will be presented in Chapter 4.

3.2.3 4-Year-Full-History sample

In the Jan1996-Live sample, the majority of the dropouts were Closed Good, which means the customer closed the account voluntarily while on good behaviour. As time progressed extra data became available, it was felt that since once a customer left the portfolio it would serve no interest to the RBS as it would not show up in the business book. Furthermore, the number of dropouts were minute when compared to the sample size, and they did not have a full set of account history, its omission would not cause a significant change in the results. The same argument applies when further analysis and illustrations were based on the Movers(2) sample alone in the following chapters. As a result a new 4-Year-Full-History sample was developed. This particular sample was based on the Jan1996-Live sample, but with the time horizon extended, so that the sample had a time horizon of 49 months from January 1996 to January 2000. The sample had identical accounts as its parent sample but it did not include any dropouts on this occasion. Thus the identical accounts from Jan1996-Live sample surviving beyond December 1998 till January 2000 stayed, and those exited the system during the period from January 1998 to January 2000 were filtered out.

3.2.4 Movers(2) and Stayer(2)

The Mover-Stayer population heterogeneity was again implemented on the 4-Year-Full-History sample by splitting the sample into two distinctive groups: Movers(2) and Stayers(2), with identical definitions applied to Movers(1) and Stayer(1). Full set of the definitions used are offered in section 3.7.

Analysis as described in Chapter 4 was performed on the Movers(2) sample. Results will be presented in Chapter 4. The transition probabilities fitted to Movers(1) and Movers(2) showed little difference, although the ones fitted to Movers(1) will reveal the terminating behaviour of accounts before closure, should one be interested.

3.2.5 Multiple Movers subsets

Results as will be shown in Chapter 4, suggested that the Movers(2) sample developed showed very weak Markov behaviour. Consequently an iterative procedure (Weiss et al, 1982) was initiated to construct a suitable state-space and/or a set of sub-populations for the data to exhibit, or at least approach, Markov behaviour. Based on the Good/Indeterminate/Bad broad category as prescribed by PROBE states definitions (Appendix 9.1), the Movers(2) sample was split into four smaller Movers groups using the total number of transitions made by the account during the study period as the basis of split: [0]-Mover, [1-to-3]-Mover, [4]-Mover, and [≥5]-Mover samples were created. [0]-Mover were those Movers who made no transition, [1-to-3]-Mover were those Movers who made one to three transitions; [4]-Mover were those Movers who made four transitions, and [≥5]-Mover were those Movers who made five or more transitions during the study period given the broad category state space. All of which had an identical time horizon as their parent sample, Movers(2). The logic behind this split will be presented and explained in Chapter 5.

3.3 Data quality and cleansing

In this thesis all the analysis was performed using either Microsoft Excel or the statistical software SAS. Overall the quality of the data on delinquency states of customers extracted from the RLS system had been good. Apart from the fact that the field which stored the delinquency data of the month end of September 2000 was not populated (Chapter 4 and 5), no missing performance data were found otherwise.

Many SAS procedures ignore missing values. For example, the PROC LOGISTIC procedure used in Chapter 6. Many cases of application details in the sample were incomplete. These cases were removed from analysis by SAS's defaults. The results in the following chapters therefore are the results computed using complete sets of data.

With large quantities of data like here, ultimate data quality is always an issue. One should consider all possibilities. The definitions of customer characteristics in section 3.7 were thought to have covered all eventualities. However, one data anomaly was discovered. There were 909 accounts which stayed in the same state other than state 1 throughout the 49 months of the study period. Figure 3.3 gives the breakdown.

PROBE state	Frequency
2	831
4	16
8	62
Total	909

Figure 3.3 Data anomalies – number of accounts stayed long term in PROBE states other than state 1

This discovery came when such cases had been made part of the Movers(2) sample but as they were less than 0.3% of the sample, it was felt they did not affect the results. They were therefore left in the sample.

3.4 Holdout samples for testing

A forecast is an estimate (or a set of estimates) about the likelihood of future events which is developed on the basis of past and current information (Pindyck and

Rubinfeld, 1998). Forecasting is the prediction of values of a variable based on known past values of that variable or other related variables (Makridakis et al, 1998). For our purpose, a "point" (or single value) forecast was a quantitative estimate of the likelihood of a customer making a transition from one delinquent state to another in the period of one calendar month. The information that may determine or explain this likelihood was embedded in the form of, as will be seen later in section 3.8, a single-equation structural model.

Forecasts can be made about future events, in our case the transition probabilities between delinquency states, by extrapolating the forecast model beyond the period over which it was fitted. The fundamental concern now was the assessment of performance of the forecast model on the data samples developed, in other words, forecast accuracy. Over-fitting is a term used to describe a forecast model which provides excellent goodness of fit to "historic" (known) data. This does not however guarantee accurate forecasts. Often, using a polynomial function of sufficient order in the fitting phase can produce exceptionally small residuals. Over-fitting a model to a data series, which is equivalent to including randomness as part of the generating process, is as bad as failing to identify the systematic pattern in the data (Markridakis et al, 1998).

This problem was overcome by dividing each data sample developed into a "Development" and a "Holdout" set temporally. The Development data set consisted of observations on the forecast and explanatory variables from the beginning periods over which the forecast model was constructed and parameters estimated. The Holdout data set usually consisted of observations on the forecast and explanatory variables from the end of the data series, which were withheld from the model fitting phase. This is called out of sample testing. This was used on the Movers(1) sample. Forecasts were made and compared with the observed values of the forecast variable in the Holdout data set. Since the observations of the forecast or the explanatory variables from the Holdout data set were not used in building the model, the forecasts produced allowed genuine forecast accuracy assessment. In this case, the forecast variable was the probability of a particular delinquency state transition, or simply transition probability. And the explanatory variables were those variables that might determine or explain the changes in the forecast variable; in this case this information would be related to the economy. The criterion for assessment chosen was percentage error, which is the difference between the forecast and actual value of the forecast variable expressed as a percentage of the actual value of the forecast variable.

An alternative approach was to test the results on accounts which were not part of the development sample at all. This is called a separate holdout sample. There was sufficient data for us to use this approach on the Movers(2) and its daughter subsets.

The evaluation of the Movers(2) sample and its daughter multiple subsets were done on a separate holdout sample. The accuracy of the forecast model was assessed using matched data that were collected after the data samples developed were all consumed in model construction and parameter estimation. This means all 49 months of delinquency (from January 1996 to January 2000) data were committed to constructing an empirical forecast model (i.e. Development set). The values of the forecast and explanatory variables in the Holdout set were not known at the model fitting stage. The RBS provided on going access to their database, one could consequently concentrate efforts on working on the model while being able to return for more data at a later date. Moreover it was a good way of telling whether the forecast model is robust in time, since the future dynamics of the forecast model, in other words the future behaviour of the RBS MTA portfolio, was the subject of interest.

3.5 Samples developed

Figure 3.4 summarises what had been achieved in sections 3.2 and 3.3, the samples developed for the analysis in Chapter 4 were Movers(1), Movers(2). As for the analysis in Chapter 5 and 6: [0]-Mover (for Chapter 6 only, not committed to analysis in Chapter 5, see Chapter 5), [1-to-3]-Mover, [4]-Mover, and [≥5]-Mover, all of which originated from Movers(2).

Samples	Size of Development set	Study period	Time span in Study period	Size of Holdout set	Holdout period	Time span in Test period	Holdout method
Movers(1)	455576 cases	January 1996 to July 1998	30 months	455576 cases	July 1998 to December 1998	6 months	out of sample
Movers(2)	315591 cases	January 1996 to January 2000	49 months	303922 cases	January 2000 to January 2001	13 months	separate sample
[0]-Mover	not committed to analysis	January 1996 to January 2000	49 months	not committed to analysis	January 2000 to January 2001	13 months	separate sample
[1-to-3]-Mover	82639 cases	January 1996 to January 2000	49 months	78600 cases	January 2000 to January 2001	13 months	separate sample
[4]-Mover	23482 cases	January 1996 to January 2000	49 months	22537 cases	January 2000 to January 2001	13 months	separate sample
[≥5]-Mover	69425 cases	January 1996 to January 2000	49 months	65572 case	January 2000 to January 2001	13 months	separate sample

Figure 3.4 Samples developed for analysis and testing.

The actual values of the forecast variable in the Holdout data set for Movers(2) and its daughter multiple subsets were collected after all the observations in the Development data set were committed to analysis. Inevitably some accounts did not survive through to the end of the test period, these were removed from the Holdout set. For example, of the original 315591 cases in the Movers(2) sample, 96% were retained through to the end of the test period.

3.6 State-Space definition

The major participant of this thesis from the RBS was the department of Risk and Lending Systems (RLS).

RLS is a retail credit unit in the RBS. It is principally involved with the development and monitoring of effective credit assessment and monitoring systems for the retail bank. Inherent to this there is a great deal of management information involved, both for the department and the retail bank as a whole.

The RLS provided a set of definitions and descriptions of the account performance measures used in their database systems, PROBE (Appendix 9.1). PROBE account states definitions formed the boundary, or the state-space, within which an account is allowed to manoeuvre according to its performance. The PROBE state is calculated for each account at the month end, taking the account's performance in the last three months into consideration. It reports the account performance at a detail level with 13 distinctive states with increasing severity of delinquency, and at a broad level with three collective categories, Good/Indeterminate/Bad. This particular set of PROBE state definitions applied only to MTA accounts. For confidentiality reasons the description of each state is omitted. The 13 detailed states though represent increasing severity of delinquency, the degree of severity however does not follow exactly the numerical sequence of the states. Some transitions are theoretically prohibited and did not appear in the data , for example for state 0 the following transitions are not possible $0 \rightarrow 7$, $0 \rightarrow 8$, $0 \rightarrow 11$, $0 \rightarrow 12$ (see Appendix 9.3 for other states). States 10, 11 and 12 are absorbing states. Other transitions between pairs of states did appear in the data set even though by the definition of the states they should not have been possible. On discussion with bank experts these were not prohibited as the transitions occurred because of bank accounting practice. In the following chapters only the non-zero transitions were included in analysis.

From here onwards in this thesis, the term "state" refers to the detailed level 13 distinctive states (0, 1, 2, ..., 12) as defined in PROBE; and the term "status" refers to the 3 collective categories (Good(G) = {state 1, 2, 10}, Indeterminate(I) = {state 0, 3, 4, 5, 6}, Bad(B) = {state 7, 8, 9, 11, 12}) that are defined in PROBE at broad level, unless stated otherwise.

Thus the transition matrix, **P**, based on the 13 PROBE states at a particular time t, would have the following form, Figure 3.5. The entries depict the probability of an account making a transition from one state to another in the period of t. Instead of probability, the entries could be the transition frequency, $N_{ij}(t)$. This information is equally useful, since the natural estimation of $P_{ij}(t)$ is $N_{ij}(t)/N_i(t)$.

$$
P(t) = \begin{array}{c|ccccc|ccc|}
 & 0 & 1 & 2 & \cdots & 9 & 10 & 11 & 12 \\
\hline
0 & & & & & & & & \\
1 & & & & & & & & \\
2 & & & & & & & & \\
\vdots & & & & P_{ij}(t) & & P_{i10}(t) & P_{i11}(t) & P_{i12}(t) \\
9 & & & & & & & & \\
\hline
10 & 0 & 0 & 0 & \cdots & 0 & 1 & & \\
11 & 0 & 0 & 0 & \cdots & 0 & & 1 & \\
12 & 0 & 0 & 0 & \cdots & 0 & & & 1 \\
\hline
\end{array}
$$

Figure 3.5 Transition matrix P (state space) based on the detailed 13 PROBE states

This transition matrix **P**(t) can be partitioned into four distinctive parts, each describes the probabilities of a set of particular transitions. The bottom right is an identity matrix, **I**, with 1's on the diagonal and zeroes elsewhere. The bottom left is a zero matrix, **0**. Together they describe once an account is closed it cannot make a transition to another state In other words, these accounts exited the system. The top left of the matrix , **Q**, describes the transitional behaviour of those accounts which remained open. The top right, **R**, describes the likelihood of an account termination from a particular state. It is the **Q** and **R** matrix for the Movers(1) sample, and **R** matrix for the Movers(2) sample (and its daughter multiple subsets), that are of interest for the purpose of this thesis.

$$
P(t) = \begin{array}{c|ccc|}
 & G & I & B \\
\hline
G & & & \\
I & & P_{ij}(t) & \\
B & & & \\
\hline
\end{array}
$$

Figure 3.6 Transition matrix P (state space) based the broad 3 PROBE status

Figure 3.6 depicts the state space based on the broad Good(G), Indeterminate(I), and Bad(B) status. In section 4.2 and 5.1 explanations will be put forward as to why it was necessary to use the reduced state space.

3.7 Characterisation of customers

In order to implement the Mover-Stayer population heterogeneity into the samples developed, distinctive groups of accounts were defined on the Jan1996-Live and the 4-Year-Full-History samples. The definitions were as follows:

STAYER were those accounts which were alive, active and stayed current (PROBE state = 1) throughout the time span of the study period.

INACTIVE were those accounts which were alive and stayed inactive throughout the time span of the study period.

CLASSIFIED were those accounts which were alive and stayed in the recovery state throughout the time span of the study period.

MOVER were those accounts which were alive, active and not STAYERS, not INACTIVE and not CLASSIFIED throughout the time span of the study period.

By definition, Movers deduced from the Jan1996-Live sample included those closed (good or bad) or written-off accounts, i.e. dropouts (section 3.2), which might have stayed current/inactive/classified otherwise. Inactive and Classified accounts must have a full history in order to be defined so.

Figure 3.7 shows the distribution of each distinctive group in the samples. These groups capture almost all types of account behaviour, under normal circumstances. Therefore, Figure 3.7 is a dissection of the MTA portfolio as of the month of January 1996 according to the individual behaviours over the study period. Given the definitions the Stayer to Mover ratio is approximately one to one in both samples.

Figure 3.7(a) Jan1996-Live sample

Group	Frequency	Percentage (%)	
Stayer(1)	359730	43.3	
Inactive(1)	13419	1.6	
Classified(1)	1199	0.1	
Mover(1)	455576	54.9	(note: included Dropout)
Total	829924	100	

Figure 3.7(b) 4-Year-Full-History sample

Group	Frequency	Percentage (%)
Stayer(2)	323176	49.7
Inactive(2)	10600	1.6
Classified(2)	754	0.1
Mover(2)	315591	48.5
Total	650121	100

Figure 3.7 Distribution of distinctive groups in the (a) Jan1996-Live and (b) 4-Year-Full-History samples

The ratio is slightly lower in the Jan1996-Live sample (0.79), Figure 3.7(a), because Movers(1) included Dropouts, the ratio is more comparable to that of the 4-Year-Full-History sample (1.02), Figure 3.7(b), when Dropouts are removed from the calculation. When Dropouts are removed, the ratio in the Jan1996-Live sample is slightly higher than that of the 4-Year-Full-History sample (1.18). This is expected, as the study period in the former lasted only 36 months and the latter lasted 49 months. The requirement to be a Stayer in the latter is more stringent. This particular result suggests RBS has a stable depository base for this particular portfolio. Second, this particular portfolio was largely active. This means the RBS can earn interest payments should customers fall delinquent as well as making use of the customers' deposits as investment funds. One intriguing fact emerged from this dissection. Though dwarfed by other behavioural groups, the RBS had let a very small number of bad accounts remained classified and unsettled indefinitely. This might be a result of some operation procedures on account closures, nevertheless once an account has been classified its movement will be dictated by RBS' business policies on recovery and collection. The Stayer to Mover ratio is likely to rise since a simplest yet the strictest definition of a Stayer had been used here.

The Mover-Stayer definitions used here were different from the ones used by Frydman et al (1985), section 2.2. It is because a customer cannot stay indefinitely in one state unless he is current. As for Movers, it will be shown in Chapter 4 that they did not follow a stationary, First Order Markov Chain.

3.8 Matching economic indicators to transition probabilities

To summarise what had been achieved so far, given the target data sets, the holdout methods and the state space, there were two Movers samples of interest. Movers(1) (originated from Jan1996-Live sample) and Movers(2) (originated from 4-Year-Full-History sample). Their differences in size, span of the study period and the holdout method used were summarised in Figure 3.4. The Jan1996-Live sample represents the "whole" account population. It explains the behaviour into defaults and closures. But once a customer exited the system one cannot see the whole account history throughout the study period. On the other hand, the 4-Year-Full-History sample distinguishes between Movers and Stayers who remained active throughout. The extended study period and the filtering of defaulting accounts will hopefully make it clearer what factors effect the volatility of essentially "good" accounts.

In the following sections, how the functional dependency of transition probability on economic indicators was established will be illustrated and explained.

3.8.1 Time Series Decomposition

A time series is a set of sequential historical data observed periodically over time. The length between two consecutive observations of time series in businesses can be typically daily, weekly, monthly, quarterly and yearly. In RBS case, PROBE states on MTA accounts are determined monthly. The basic idea of time series decomposition is to break up a time series into its constituents hence improve understanding the underlying pattern and structure in the data series and distinguishing it from randomness. There are two major components that make up and characterise a time series:

Trend-Cycle component, $Tr(t)$, represents the long term changes in the level of a time series. Seasonal component, $Sn(t)$, represents the periodic fluctuations to the level of a time series due to seasonal factors such as month of the year, timing of a season, and so on, that is, pattern which repeats itself over regular intervals of time. The Error term, $Er(t)$, represents the randomness that produced the series, and is the difference between the forecast, in other words the combined effect of the trend-cycle and seasonal components, and the actual value of the observation. A time series can be expressed mathematically, substituting $P_{ij}(t)$ for our purpose:

$$\mathbf{P_{ij}(t) = f(Tr_{ij}(t), Sn_{ij}(t), Er_{ij}(t))} \qquad \textbf{Equation 3.1}$$

3.8.2 Decomposition Model

The exact functional form of Equation 3.1 depends on the decomposition model assumed. And the decomposition method used also depends on the model. There are several forms commonly used but the simplest are the Additive (Equation 3.2) and Multiplicative (Equation 3.3), or Logarithmic (Equation 3.4) if one is indecisive between additive or multiplicative decomposition.

$$\mathbf{P_{ij}(t) = Tr_{ij}(t) + Sn_{ij}(t) + Er_{ij}(t)} \qquad \textbf{Equation 3.2}$$

$$\mathbf{P_{ij}(t) = Tr_{ij}(t) * Sn_{ij}(t) * Er_{ij}(t)} \qquad \textbf{Equation 3.3}$$

$$\mathbf{\log P_{ij}(t) = Tr_{ij}(t) + Sn_{ij}(t) + Er_{ij}(t)} \qquad \textbf{Equation 3.4}$$

There was obviously a choice here as to which model to assume. It was necessary to explore the data samples before making any decision. Time series charts were produced to visualise the features exhibited in the data. A time series chart is a graph of the measurement of interest observed against time over which the observations were made. It is a simple graphical summary but an effective way to "get a feel" of the data observed before any assumptions or analysis is undertaken. Bearing in mind the number of observed $P_{ij}(t)$ series increases exponentially, m^{n+1} (where n is the order of the Markov Chain), a couple were selected here for illustration. The selected $P_{ij}(t)$ series are either dominated (e.g. $N_{11}(t)$ alone constitutes approximately 50 to 60% of all transition frequencies in Movers(2) sample) ones or ones which should reflect the whole spectrum of delinquency. The selected $P_{ij}(t)$ series are $P_{11}(t)$ and $P_{78}(t)$. $P_{11}(t)$ is the probability of transition from being current (PROBE state 1) at time t to current (PROBE state 1) again at time t+1. $P_{78}(t)$ represents the other end of the delinquency spectrum, it is the probability of transition from being over 2 months delinquent (PROBE state 7) at time t to being more than 3 months delinquent (PROBE state 8) at time t+1.

According to Pegels' (1969) classification of forecasting scenarios, the patterns exhibited in the observed $P_{11}(t)$ series was identified to conform to a linear model with additive trend-cycle and seasonal components. In Figure 3.8(a) the observed $P_{11}(t)$ series shows an apparent downward trend in the level of the series, with periodic fluctuation zigzagging across. The classical time series decomposition procedures for an additive time series (Equation 3.2) as described by Makridakis et al (1998) was performed to separate the series into its elements. The basic idea was to first remove the trend-cycle component before isolating the seasonality and finally determine the randomness in the series. A centred 12 point moving average (or 12 MA smoother) was computed for the observed $P_{11}(t)$ series. This was the trend-cycle $Tr(t)$ component for the $P_{11}(t)$ series. It was plotted and superimposed with the observed series in Figure 3.8(a), which confirms the overall downward trend in the level of the series The observed $P_{11}(t)$ series was then de-trended, $P_{11}(t) - Tr_{11}(t) = Sn_{11}(t) + Er_{11}(t)$, and the seasonal index for a month was computed by taking an average of the de-trended values for a given month. The seasonal indices obtained were normalised assuming they were constant over time. In this case the 12 monthly indices should sum up to zero in a calendar year meaning they should not have any effect on the overall level of the series. The seasonal indices for the $P_{11}(t)$ series were stringed together and plotted in Figure 3.8(b). Figure 3.8(b) shows the seasonal indices of the $P_{11}(t)$ series have a negative effect on the overall level in February, May, August and November These months correspond to time just prior to the Easter break, summer holidays, start of a new academic year and Christmas respectively, when consumers are traditionally spending more than usual. The November index can reduce the level of the $P_{11}(t)$ series by as much as 0.02. The error term, Er(t), that represents the randomness in the $P_{11}(t)$ series was the residual between the observed value and the sum of the trend-cycle and seasonal components, $P_{11}(t) - Tr_{11}(t) - Sn_{11}(t) = Er_{11}(t)$, Figure 3.8(c). It shows irregular runs of positive and negative values. Makridakis et al (1998) argued that serial uncorrelation in the error terms is often not the case, and the decomposition approach may have theoretical weakness from a statistical point of view. Nevertheless it serves well as a

tool for providing and understanding an insight into the behaviour of a time series through graphical display.

Identical treatments were performed on the $P_{78}(t)$ series.

Here the $P_{78}(t)$ series shows an apparent upward trend in the level of the series, Figure 3.9(a). The seasonal fluctuations were more eccentric and irregular especially towards the end of the study period. The seasonal indices for the $P_{78}(t)$ series show a positive influence on the level of the series where the seasonal indices of the $P_{11}(t)$ series show the opposite, Figure 3.9(b). The error terms again show irregular runs of positive and negative values, Figure 3.9(c).

Figure 3.8(a)

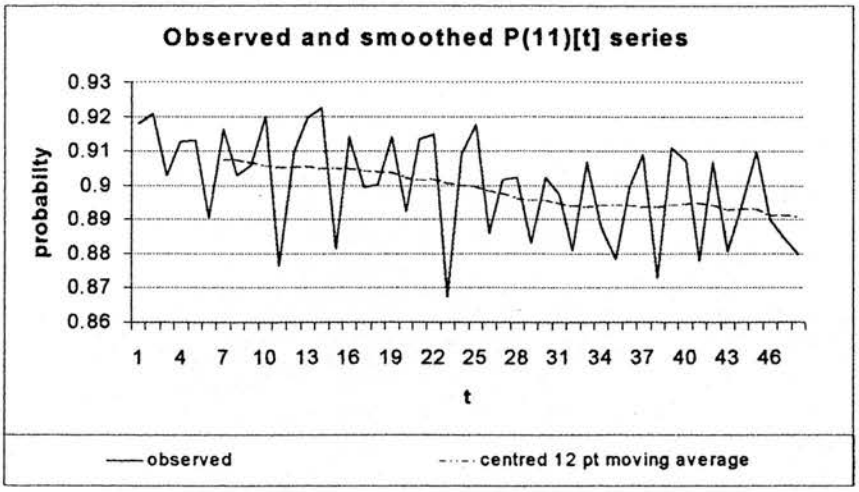


Figure 3.8(b)

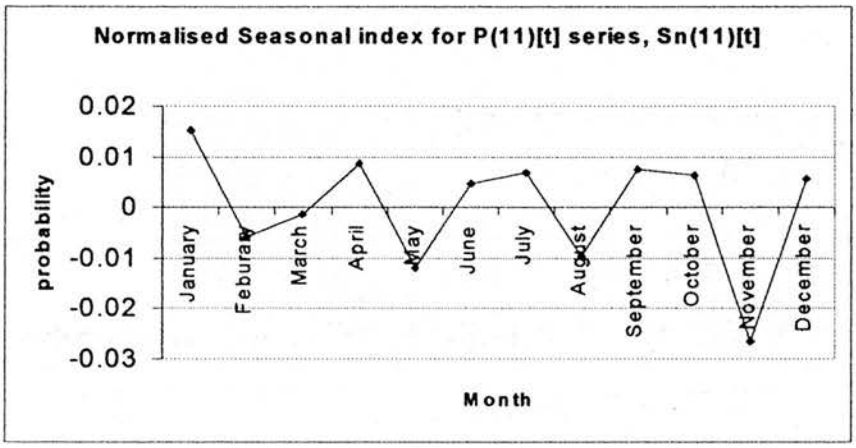


Figure 3.8(c)

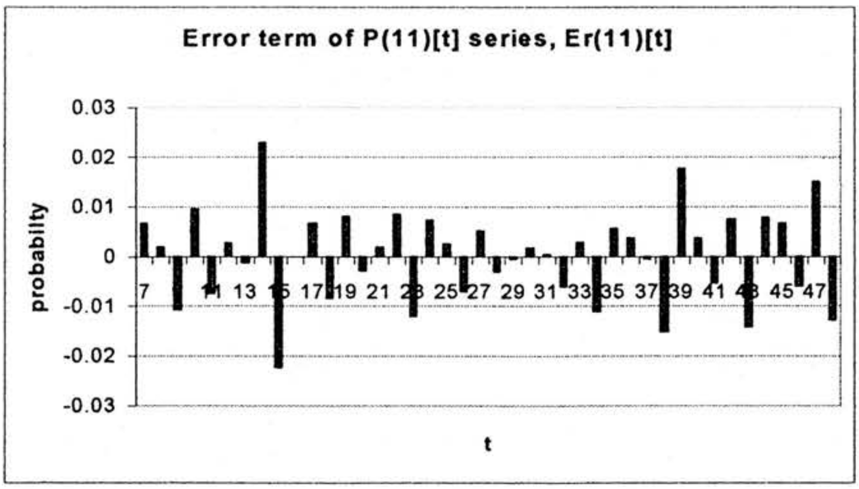


Figure 3.8 Time series plot and decomposition for $P_{11}(t)$ series of Movers(2) sample

Time series decomposition was performed to series, though not all, other than $P_{11}(t)$ and $P_{78}(t)$. And they were not displayed here. A decision was made at this point based on the Author's judgement that $P_{ij}(t)$ series observed were largely additive, composed of a trend-cycle and a seasonal component. There were two main reasons. First it was not sensible nor practical to decompose all $P_{ij}(t)$ series given the time allowed for producing a thesis, since the number of $P_{ij}(t)$ series in this case was large. Moreover time series decomposition is only a preliminary tool for an understanding and exploring the data, there was a large amount of analysis which followed. Second, $N_{11}(t)$ was by far the most dominating transition in terms of numbers. If any assumptions were to be made then they should be based on the majority.

Figure 3.9(a)

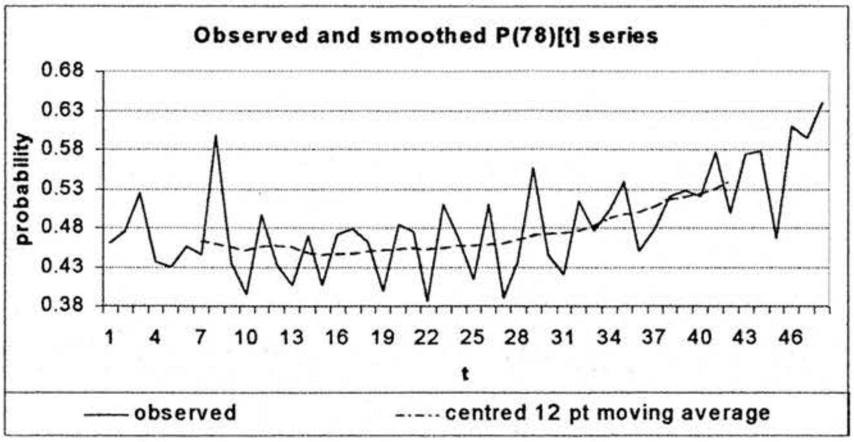


Figure 3.9(b)

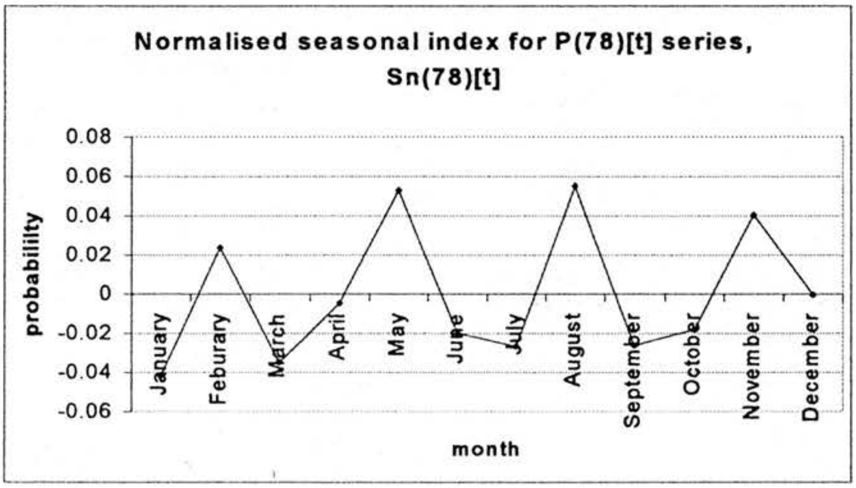


Figure 3.9(c)

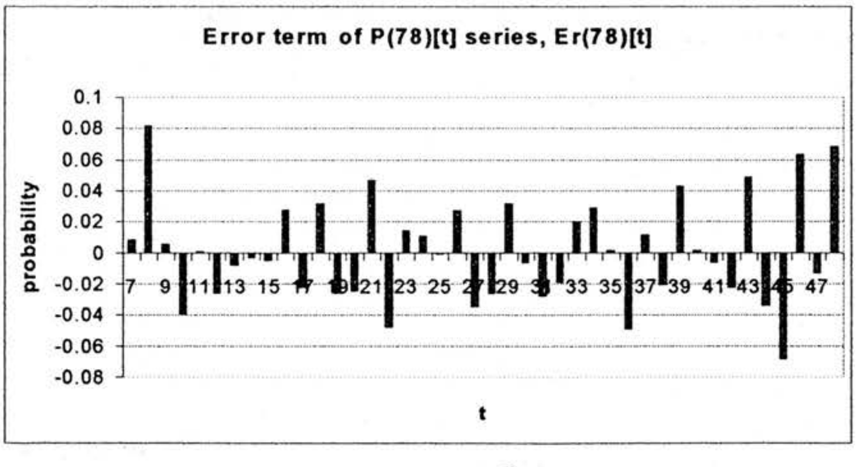


Figure 3.9 Time series chart and decomposition for $P_{78}(t)$ series of Movers(2) sample

3.8.3 Forecast model and choice of economic indicators

In section 3.8.2 it was showed that the transition probability time series, $P_{ij}(t)$, was composed of a trend-cycle component, a seasonal index and an error term, which when added together make up the observed series. In this section it will be showen how point forecasts would be made on the observed series. The seasonal index was relatively easy to deal with since it was assumed constant over time. One needed to have enough data to cover a full calendar year. In this case there was ample. The error term simply cannot be predicted or simply is forecasted as zero for an additive model (or forecasted as one for a multiplicative model). This left the trend-cycle component, Tr(t). Makridakis et al(1998) argued that Tr(t) is actually made up by a separate trend and cycle component, though the distinction is purely artificial. The Tr(t) for $P_{11}(t)$ and $P_{78}(t)$ series in Figure 3.8(a) and 3.9(a) did not show cyclic behaviour as in many typical sales data. Though in the $P_{11}(t)$ case, Figure 3.8(a), two plateau areas had been revealed, first around t = 12 to 18, then t = 32 to 40. So this distinction can be ignored here. This implied a simple, parametric, linear, single-equation regression model would be sufficient for forecasting $Tr_{ij}(t)$. And the additive nature of the trend-cycle, seasonal and error components would produce the final estimated P_{ij}. In this section the choice of explanatory variables and goodness of fit of the forecast model will be discussed. The $P_{11}(t)$ and $P_{78}(t)$ series from the Movers(2) sample again are used, for consistence, to illustrate. A linear regression model is a model where changes in a dependent variable can be explained by changes in some explanatory variables. The relationship between the dependent and explanatory variables is linear. In practice, a complete independence between explanatory variables is rare. Highly correlated explanatory variables however will make interpretation of the regression coefficients difficult. A regression model was chosen because it is well studied and practised. Furthermore it can accommodate, if necessary, numerous explanatory variables. Finally, the estimation procedures of parameters associated with explanatory variables, the least squares method, are readily incorporated and found in many spreadsheet or other analytical software packages.

Figure 3.10(a)

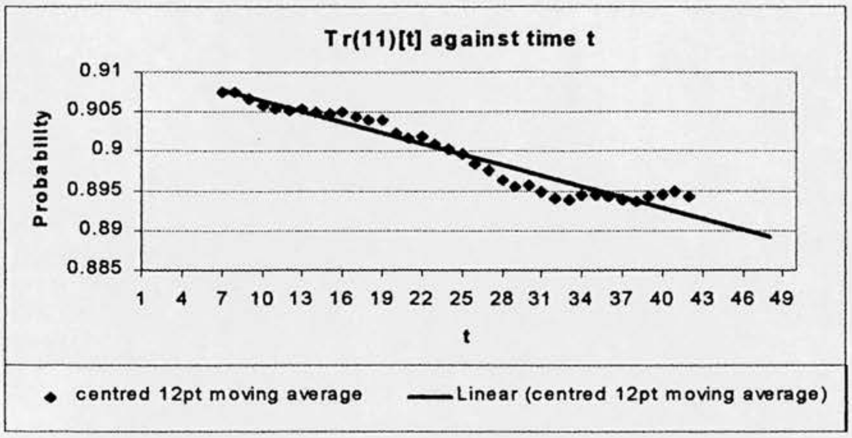


Figure 3.10(b)

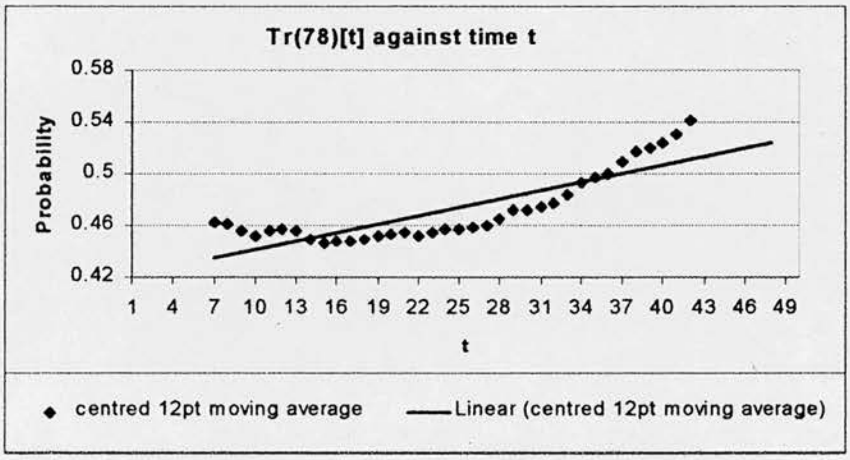


Figure 3.10 Scatter charts of (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$ series on t

In this research, one wanted to study the evolution of credit behaviour over time. One was also interested in how credit behaviour reacted to changes in the external environment. There is a pool of economic indicators that are readily available in the public domain. But there were conditions to be met when choosing one in this thesis. The selected indicators needed to be temporally compatible with the monthly account performance data extracted. Each explanatory variable should bring new information without inducing severe multicollinearity into the model. The indicators chosen can reflect the changes in the macro- (changes that are shared by all companies in the economy) or micro- (changes that apply only to individual company) economy, or possibly both.

Figure 3.11(a)

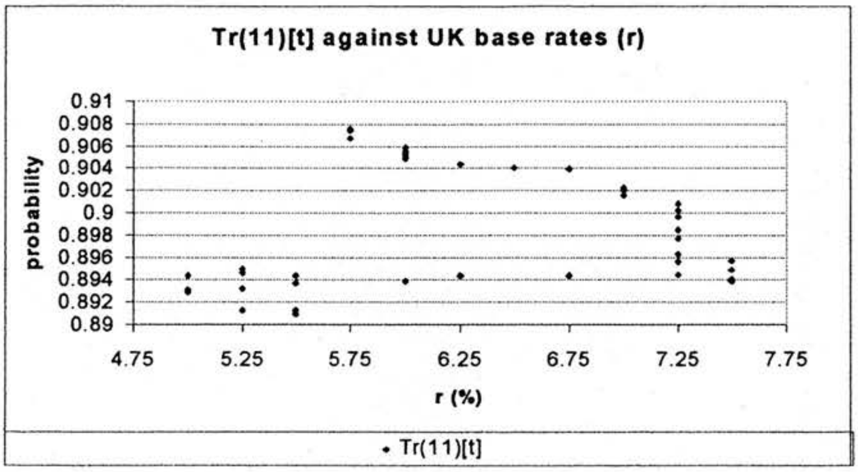


Figure 3.11(b)

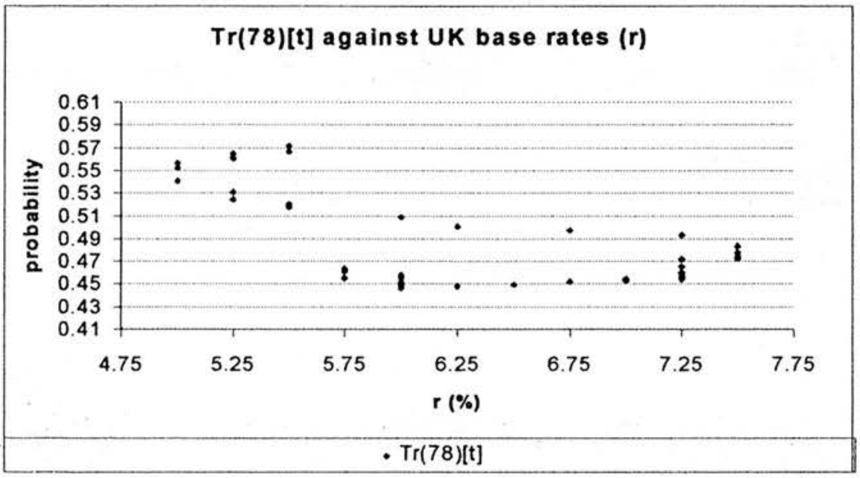


Figure 3.11 Scatter charts of (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$ series on r

The choice made here was the UK retail bank's base rates (r) (Bank of England) complied by the Bank of England. The reason was two-fold. First it is the rate that applies to all lenders in the UK. Second it is the rate which would determine the interest margins charged to customers on different credit products by individual banks. Interest rates are traditionally seen as the cost of borrowing. It is the compensation demanded by lenders of borrowers for the use of borrowed funds over a period of time. In other words it is the "price of credit", which could serve as a price index or demand indicator for all credit products (which in turn is a good indicator of the demand for items of substantial purchase, like household appliances, motors, holidays, etc.). Therefore the base rate is sufficient to reflect the changes taking place in the macro-economy and a good choice of explanatory variable here. Figure 3.12 displays the changes in UK retail banks' base rates (r) over the study period.

Figure 3.10(a) and 3.10(b) display the scatter plots of $Tr_{11}(t)$ and $Tr_{78}(t)$ against t, respectively. Figure 3.13(a) and (b) show the ANOVA and regression outputs of regressing $Tr_{11}(t)$ and $Tr_{78}(t)$ on t alone respectively. The F-statistic in both cases confirmed that at 1% significance level there is a significant overall regression effect between the dependent variables and t. The curvature of the $Tr_{78}(t)$ series, Figure 3.10(b), can be better fitted with a polynomial function of suitable order. It was not attempted because of the argument put forward at the end of section 3.8.2 - the fact that there were other explanatory variables to be fitted, and the ease of use of a linear regression model. It was assumed that the trend-cycle component of all the series extracted can be largely forecasted utilising a linear regression model.

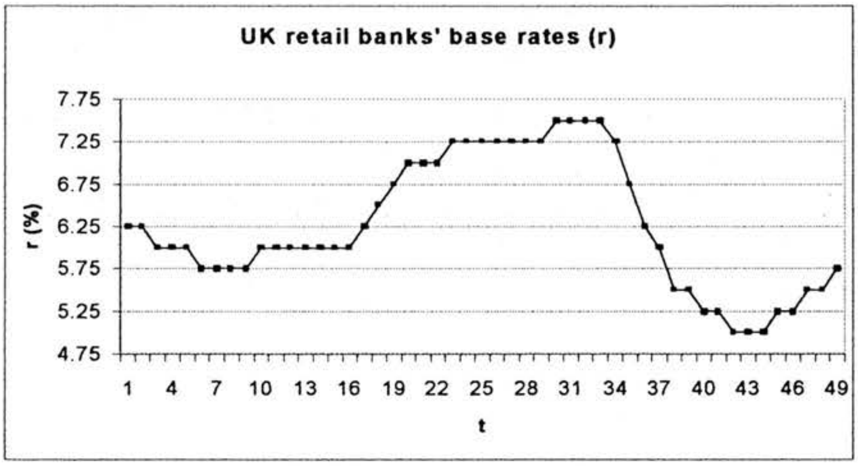


Figure 3.12 UK base rate r over the study period

Figure 3.13(a) Regressing $Tr_{11}(t)$ on t

Dependent Variable: $Tr_{11}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00081	0.00081	485.603	0.0001
Error	34	5.7*10e-5	1.67*10e-6		
Total	35	0.00086		$R^2 = 0.9346$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	34	0.910974	5.5*10e-4	1654.848	0.0001
t	34	-0.000456	2.1*10e-5	-22.036	0.0001

Figure 3.13(b) Regressing $Tr_{78}(t)$ on t

Dependent Variable: $Tr_{78}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.01836	0.01836	85.08	0.0001
Error	34	0.00734	0.00022		
Total	35	0.02569		$R^2 = 0.7145$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	34	0.419521	0.00627117	66.897	0.0001
t	34	0.002174	2.36*10e-4	9.224	0.0001

Figure 3.13 Results from regressing (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$ on t

Figure 3.11(a) and 3.11(b) display the scatter plots of $Tr_{11}(t)$ and $Tr_{78}(t)$ against r, respectively. Figure 3.14(a) and (b) show the ANOVA and regression outputs of regressing $Tr_{11}(t)$ and $Tr_{78}(t)$ on r alone respectively. The null hypothesis that there was no regression effect could not be rejected at the 5% significance level for $Tr_{11}(t)$. This null hypothesis was rejected at the 1% significance level for $Tr_{78}(t)$.

Figure 3.14(a) Regressing $Tr_{11}(t)$ on r

Dependent Variable: $Tr_{11}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00004	0.00004	1.771	0.1921
Error	34	0.00082	0.00002		
Total	35	0.00086		$R^2 = 0.0495$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	34	0.909155	0.00707168	128.563	0.0001
r	34	-0.001443	0.00108410	-1.331	0.1921

Figure 3.14(b) Regressing $Tr_{78}(t)$ on r

Dependent Variable: $Tr_{78}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00464	0.00464	7.501	0.0097
Error	34	0.02105	0.00062		
Total	35	0.02569		$R^2 = 0.1807$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	34	0.570178	0.03580523	15.924	0.0001
r	34	-0.015033	0.00548902	-2.739	0.0097

Figure 3.14 Results from regressing (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$on r

Results so far suggested that base rate r alone cannot significantly explain the changes in $Tr_{11}(t)$. Regressing on t and r collectively produced a different picture because Figure 3.12 shows base rate r is a time series itself (i.e. r(t)).

Figure 3.15(a) Regressing $Tr_{11}(t)$ on t AND r(t)

Dependent Variable: $Tr_{11}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	2	0.00083	0.00042	443.18	0.0001
Error	33	3.1*10e-5	9.4*10e-7		
Total	35	0.00086		$R^2 = 0.9641$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.9181	0.00142862	642.648	0.0001
t	33	-0.000452	1.6*10e-5	-28.997	0.0001
r(t)	33	-0.001116	2.1*10e-4	-5.211	0.0001

Figure 3.15(b) Regressing $Tr_{78}(t)$ on t AND r(t)

Dependent Variable: $Tr_{78}(t)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	2	0.02404	0.01202	239.914	0.0001
Error	33	0.00165	5*10e-5		
Total	35	0.02569		$R^2 = 0.9357$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.525856	0.01043178	50.409	0.0001
t	33	0.002237	1.1*10e-4	19.676	0.0001
r(t)	33	-0.016653	0.00156365	-10.65	0.0001

Figure 3.15 Multiple regression results from regressing (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$ on t and r(t) collectively

For both $Tr_{11}(t)$ (Figure 3.15(a)) and $Tr_{78}(t)$ (Figure3.15(b)) , the results showed that $Tr_{ij}(t,r(t)) = a_{ij} + b_{ij}t + c_{ij}r(t)$ is a significant regression model at 1% significance level. Each of the explanatory variables is significantly different from zero at 1% significance level (two-sided test) in the presence of all other variables in the model. In both cases, over 90% (the correlation coefficient, R^2) of the variance in the dependent variable can be explained by the linear model with t and r(t).

The high explanatory power of the regression models can be seen visually when the $Tr_{11}(t)$ series (Figure 3.10(a)) was superimposed onto the UK base rates (Figure 3.12), over the identical time horizon, as displayed in Figure 3.16(a). The good fit of the chosen series was partly due to smoothing of the data. Figure 3.16(a) shows the $Tr_{11}(t)$ series dipped below the ($Tr_{11}(t) = a_{11} + b_{11}t$) regressed line when base rates were high, and surged above it when base rates were low. This suggested customers had difficulty remaining current when interest rates were high, and performed better when rates were low. This result was expected. The same treatment was performed on the $Tr_{78}(t)$ series, and is displayed in Figure 3.16(b). It shows the $Tr_{78}(t)$ series dipped below the ($Tr_{78}(t) = a_{78} + b_{78}t$) regressed line when rates were high and surged above the line when rates were low. This suggested customers did not want to suffer high interests payments when rates were high, and relaxed a little when rates were low. Figure 3.16(a) and (b) seem to portrait a contradictory picture, that good customers' performance declined and bad customers' performance improved when rates were high, and vice versa.

This phenomenon suggested a population homogeneity in the Movers(2) sample, i.e. those loyal good customers and those customers not bad enough to be written off revolving around. Nevertheless these results suggested t and r(t) together could provide a good predictive model.

No other economic indicators were considered for two reasons. There are relationships between other economic indicators and base rate (r). For example one would expect mortgage payment as a percentage of income and base rate to be highly collinear. This would induce multicollinearity into the regression model. Many economic figures are compiled quarterly or annually, like GDP, inflation; making them unsuitable in this case.

As a result, the final functional form of the forecast model had been decided and settled as follows:

$$\mathbf{Tr_{ij}(t,r(t)) = a_{ij} + b_{ij}t + c_{ij}r(t)} \qquad \textbf{Equation 3.5}$$

The advantage of this form is that the impact of time and interest rate is clear. The disadvantage is that one needs to force the result to stay between 0 and 1. An alternative form would be log $Tr_{ij}(t,r(t)) = a_{ij} + b_{ij}t + c_{ij}r(t)$. This has the advantage that $Tr_{ij}(t,r(t))$ is guaranteed to be between 0 and 1. But the effect of time and interest rate is not clear, since the normalising constant will be a function of b_{ij} and c_{ij} for all other j.

Figure 3.16(a)

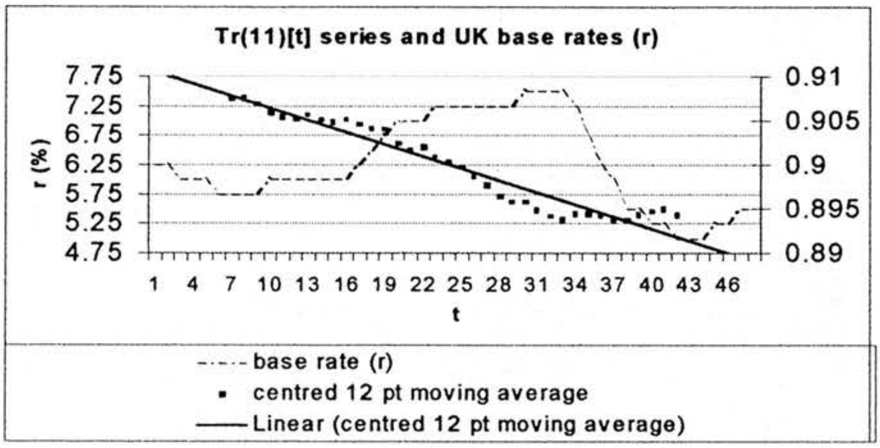


Figure 3.16(b)

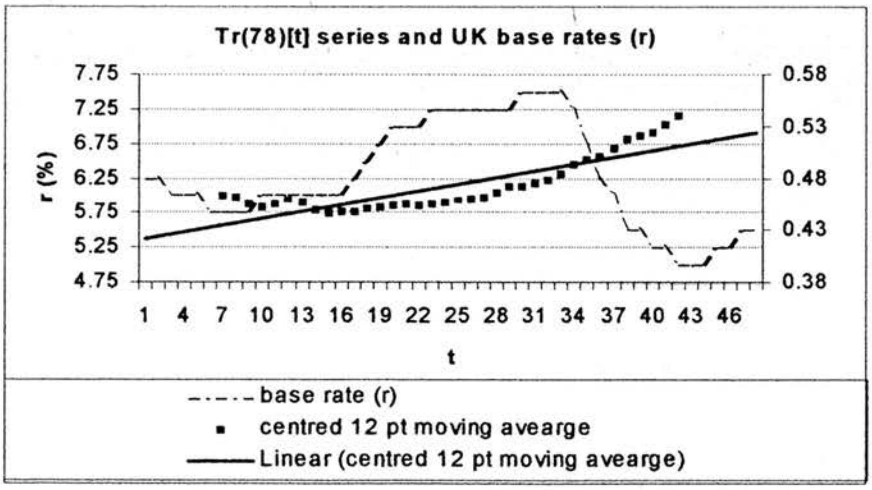


Figure 3.16 (a) $Tr_{11}(t)$ and (b) $Tr_{78}(t)$ series superimposed onto UK base rates r(t)

It has been demonstrated in this section that Equation 3.5 worked well so far to one transition probability series at a time. However, the matter was complicated by the fact that there is more than one destination for a given performance state, and all transition probabilities for a given performance state must sum up to one. A whole new mechanism and procedures were necessary to estimate the value of the parameter associated with each explanatory variable. This will be presented in Chapter 4.

3.9 Conclusion

In this chapter we described how the samples for analysis were constructed. The state-space in which each case in the samples developed was allowed to manoeuvre was presented. In order to introduce the Mover-Stayer population heterogeneity, definitions were offered and explained. Transition probabilities series were extracted and then broken down into its elements. The decomposition approach was merely a tool for understanding what constitutes the time series extracted and what was the underlying generating process. It was shown that a transition probability series was composed of three additive components: a trend-cycle, a seasonal and an error component. A linear single-equation forecast model based on multiple regression was proposed to estimate the trend-cycle component. In order to obtain an optimal model, choices of explanatory variables were presented and explained. Throughout the chapter, rationale and justifications were put forward and argued to the decisions and choices made, assumptions made, and the methods used. These will form the basis for discussion in the conclusion chapter (Chapter 7).

4. Model for General Population

4.1 Why Markov Chain models would be useful – The Model

This thesis seeks to extract a valid empirical model which can extrapolate the future credit behaviour from the past, and the future credit behaviour can be explained by changes in some economic indicators. A model based on Markov Chains would be useful because one could incorporate past history into the estimation of the future, and in this case, add economic indicators into the model as explained in Chapter 3. One wanted to prove that the CDT assumptions were not ideal for the given data. One also tried to see what are the relative advantages of modification of the benchmark model (i.e. the CDT model). The foundation and tests of the Markov Chain were laid out by Anderson and Goodman (1957). Details of the analysis and tests carried out on the Movers(2) sample will be discussed in the following sections. Movers(2) were those customers who remained active throughout the study period. This means each case in the sample had a full set of account history. The extended study period and the filtering of closed accounts will depict the behaviour of essentially "good" accounts, section 3.8.

The Model:

Let account performance/delinquency states (PROBE states) be $i = 0, 1, 2, \ldots, m$.

Let the month end at which the value i was observed be $t = 1, 2, \ldots, T$, where T is the end of the study period.

Let $N_{ij}(t)$ be the number of accounts being in state j at time t+1 given being in state i at time t (i.e. the transition frequency) for $i, j = 1, 2, \ldots, m$; $t = 1, 2, \ldots, (T-1)$.

Let $N_{ij} = \sum_{t=1}^{T-1} N_{ij}(t)$

Let $N_{gh...ij}(t)$ be the number of accounts being in states g, h, ... i, j at respective times t-n+1, t-n+2, ..., t, t+1 for g, h, i, j = 1, 2, ..., m; t = 1, 2, ..., (T-n); where n is the order of the Markov Chain.

Let $N_{gh...ij} = \sum_{t=1}^{T-n} N_{gh...ij}(t)$

Let $P_{ij}(t)$ be the transition probability of an account being in state j at time t+1, given being in state i at time t, for i, j = 1, 2, ..., m; t = 1, 2, ..., (T-1).

Let $P_{gh...ij}(t)$ be the transition probability of an account being in states g, h, ... i, j at respective times t-n+1, t-n+2, ..., t, t+1 for g, h, i, j = 1, 2, ..., m; t = 1, 2, ..., (T-n); where n is the order of the Markov Chain.

4.2 Test for Markovity

The simplest form of a Markov Chain is that of a First Order Markov Chain. It means that the future state of a system is dependent only on the current state of the system. In terms of delinquency states, one's future behaviour at time (t+1) is dependent conditionally and solely on one's performance at time t. Higher order Markov Chains are where the future state of a system depends conditionally on two or more previous states. In terms of delinquency states, one's future behaviour at time (t+1) depends conditionally on one's performance at time t (first order) and (t-1) (for a second order Chain), and (t-2) (for a third order Chain), and (t-3), ..., and so on. A first order Markov Chain had been assumed without rigorous testing in many papers cited in Chapter 2. The first test to be carried out in order to apply a Markov Chain here was to test its suitability for the data extracted.

The hypothesis that the Markov Chain is of a given order for the given samples is tested using Chi-square tests. Figure 4.1 displays the m×m contingency table. This has $(m-1)^2$ degrees of freedom (assuming stationary chains for now, or $T(m-1)^2$ for non-stationary chains). The null hypothesis is that the underlying status (Good(G), Indeterminate(I), Bad(B)) transition process is a Markov Chain of n-th order, H_o: $P_{gh\ldots ij} = P_{h\ldots ij}$ $(i, j = 1, 2, \ldots, m)$. This was tested against the alternative hypothesis that the process was a (n+1)-th order Markov Chain.

The Chi-square statistic appropriate to the hypothesis being tested was:

$$\chi^2_{h\ldots i} = \Sigma_{g,j,t}\, N_{gh\ldots i}(t) * (\hat{P}_{gh\ldots ij} - \hat{P}_{h\ldots ij})^2 / \hat{P}_{h\ldots ij} \quad \text{where } \hat{P}_{h\ldots ij} = \frac{\sum_{t=1} N_{h\ldots ij}(t)}{\sum_{t=0} N_{h\ldots i}(t-1)}$$

The Chi-square statistic applied to testing if the Markov Chain was first order not second order is as follows. For the transitions from state i to be Markov we calculated the χ^2 value $\sum_h \sum_j \frac{(N_{hij} - E_{hij})^2}{E_{hij}}$ where E_{hij} is the expected number of transitions from state h to i to j. $E_{hij} = N_{hi} * \hat{P}_{ij} = N_{hi}N_{ij} / N_i$ = (Row Total*Column Total)/Grand Total). So it became $\sum_h \sum_j \frac{(N_i N_{hij} - N_{hi}N_{ij})^2}{N_i N_{hi} N_{ij}}$.

To test if the Markov Chain was n-th order not (n+1)-th order, the test became:

$\sum_h \sum_j \frac{(N_{hi_1\ldots i_m j} - E_{hi_1\ldots i_m j})^2}{E_{hi_1\ldots i_m j}}$ where $E_{hi_1\ldots i_m j} = N_{hi_1\ldots i_m} * \hat{P}_{i_1\ldots i_m j} = \frac{N_{hi_1\ldots i_m} N_{i_1\ldots i_m j}}{N_{i_1\ldots i_m}}$. So it became

$$\sum_h \sum_j \frac{(N_{i_1\ldots i_m} N_{hi_1\ldots i_m j} - N_{hi_1\ldots i_m j} N_{i_1\ldots i_m j})^2}{N_{i_1\ldots i_m} N_{hi_1\ldots i_m} N_{i_1\ldots i_m j}}$$

Figure 4.1(a)

all t	f(ij)			
f(hi)	11	12	...	1m
11	N_{111}	N_{112}	...	N_{11m}
21	N_{211}	...		
⋮	⋮			⋮
m1	N_{m11}			N_{m1m}

Figure 4.1(b)

all t	f(ij)			
f(ghi)	11	12	...	1m
111	N_{1111}	N_{1112}	...	N_{111m}
121	N_{1211}	...		
⋮	⋮			⋮
mm1	N_{mm11}			N_{mm1m}

Figure 4.1 Contingency tables for testing (a) First against Second order (b) Second against Third order Markovity on reduced state space

Under the null hypothesis one tested that there was no difference in the distribution of the proportion of $N_{h...ij}$. If the null hypothesis was true for the data then it means a customer's transition into state j at time (t+1) was conditional on state i at time t regardless of his state h at time (t-1), in the case of testing for a first order against a second order Markov Chain. The state space based on the broad delinquency status (Figure 3.6) was used here instead of the fine delinquency states (Figure 3.5). It was because not only the number of possible transitions increase exponentially with m^{n+1}, many of which would be theoretically and physically impossible. This would create many sparse if not zero entries to the matrix, which would render the results unreliable. Using a reduced state space should eliminate the occurrence of zero/sparse entries.

4.3 Results of the Markovity test

Figure 4.2 summarises the results of Chi-square values from testing first and second order Markovity on the Movers(2) sample. One striking feature from the figures is the magnitude of the Chi-square values calculated. These Chi-square values were huge when compared to the critical value. As a result it can be concluded convincingly that the null hypothesis of the Markov Chain being first and second

n-th order Markovity tested	state i	calculated Chi-square	degrees of freedom (df)	critical value (df = 4,0.05)
1	G	374901	4	9.49
	I	182173	4	9.49
	B	11732	4	9.49
n-th order Markovity tested	state h and i	calculated Chi-square	degrees of freedom (df)	critical value (df = 4,0.05)
2	GG	164853	4	9.49
	IG	853	4	9.49
	BG	160	4	9.49
	GI	4265	4	9.49
	II	47664	4	9.49
	BI	166	4	9.49
	GB	109	4	9.49
	IB	486	4	9.49
	BB	7150	4	9.49

Figure 4.2 Chi-square results at state level from testing First and Second Order Markovity on the Movers(2) sample

order was rejected at 5% significance level. However, Figure 4.2 also showed the values of Chi-square calculated significantly reduced in magnitude from testing first to second order Markovity. Hence this seems to indicate that future behaviour can be predicted with certain accuracy using a Markov Chain of a suitable order alone given long enough past history are considered and incorporated.

This result justified the use of the reduced state space (Figure 3.6) in this test. It showed conclusively that the data (Movers(2)) did not show first and second order Markov behaviour. The analysis could be repeated on the full 13-state space (Figure 3.5). However 2^{nd} and 3^{rd} order chains would involve over 100 and over 1000 states respectively. Such large state spaces were ruled out because they were difficult for managers to understand and because the data for each transition probability estimate become small.

4.4 Test for Stationarity

Constant transition probabilities over time (assuming a First Order chain for now) was assumed in many papers cited in Chapter 2, even though authors had expressed doubts they had assumed otherwise. For the purpose of this thesis, a rigorous test was needed to test the validity of such a hypothesis – the assumption of constant transition probabilities through time. Chi-square tests in the form of a m×T contingency table, Figure 4.3, were used as laid out by Anderson and Goodman (1957).

Although the results from section 4.3 queried whether the chain is first order, we will consider it to be first order while we are testing for stationarity. This is because the current models used in the industry assume the chains are both first order Markov and stationary. We wish to investigate which is the more important assumption to relax first. Clearly eventually we would wish to look at non-stationary higher order Markov Chains, but this thesis is looking only at the first steps in the process.

The null hypothesis was H_o: $P_{ij}(t) = P_{ij}(t = 1, 2, \ldots, T)$. It was tested at state level given the fine state space (Figure 3.5) separately for each given i, all j and t with (m-1)(T-1) degrees of freedom. It had the following test statistic:

$$\chi_i^2 = \Sigma_{t,j}\, N_i(t-1) * (\hat{P}_{ij}(t) - \hat{P}_{ij})^2/\hat{P}_{ij} \text{ where } \hat{P}_{ij}(t) = \frac{N_{ij}(t)}{N_i(t-1)} \text{ and } \hat{P}_{ij} = \frac{N_{ij}}{N_i}$$

The joint hypothesis at chain level H_o: $P_{ij}(t) = P_{ij}$, $t = 1, 2, \ldots, T$ for all i, j and t was also tested with m(m-1)(T-1) degrees of freedom. The contingency table for testing the joint hypothesis can be constructed by stacking the contingency tables of all states i together. And the Chi square statistic for the joint hypothesis is the sum of all Chi square statistics of all states i.

$$\chi^2 = \sum_{i=1}^{m} \chi_i^2 = \Sigma_i \Sigma_{t,j} N_i(t-1) * (\hat{P}_{ij}(t) - \hat{P}_{ij})^2 / \hat{P}_{ij}$$

So to test the null hypothesis at state and chain level we calculated the Chi square values $\Sigma_{t,j} \frac{[N_{ij}(t)N_i - N_{ij}N_i(t-1)]^2}{N_i(t-1)N_iN_{ij}}$ and $\Sigma_i\Sigma_{t,j} \frac{[N_{ij}(t)N_i - N_{ij}N_i(t-1)]^2}{N_i(t-1)N_iN_{ij}}$ respectively.

Two cases of stationarity were considered: Seasonal (Figure 4.3(a)) and Trend (Figure 4.3(b)) stationarity. The values of t were sorted in ascending order in the test matrix. Seasonal stationarity is where the months share the same values of MOD(t/12) across calendar years. When grouped together in one calendar year, all the months have a common value of INT(t/12) in Trend stationarity. A rejected null hypothesis in the case of seasonal stationarity means transitions were not constant within a given calendar year (i.e. 1996, 1997, 1998, 1999). This means that transitions were due to the different times or months of the year. A rejected null hypothesis in the case of trend stationarity means that transitions were not constant over calendar years given the same month (e.g. March of 1996, 1997, 1998, 1999). This means there is a trend in the transitional behaviour.

The null hypothesis being valid is equivalent to the column distributions being identical across all T. Only non-zero and non-sparse entries were used in testing stationarity in order to conform to the requirements of a Chi-square test. Such precaution was not required when testing Markovity in section 4.2 because the data were amalgamated.

Figure 4.3(a)

Given i j	t 1	 2	 ...	 12
1	$N_{i1}(1)$	$N_{i1}(2)$	...	$N_{i1}(12)$
2	$N_{i2}(1)$	...		
⋮	⋮			⋮
m	$N_{im}(1)$			$N_{im}(12)$

Figure 4.3(b)

Given i j	t 1	 13	 25	 ...	 t+12
1	$N_{i1}(1)$	$N_{i1}(13)$	...	...	$N_{i1}(t+12)$
2	$N_{i2}(1)$	...			
⋮	⋮				⋮
m	$N_{im}(1)$				$N_{im}(t+12)$

Figure 4.3 Contingency tables for testing (a) Seasonal and (b) Trend Stationarity at state level

4.5 Results of the Stationarity test

Figure 4.4 shows the results from the Chi-square tests for seasonal and trend stationarity at individual state level for PROBE state 1 (Figure 4.4a) and PROBE state 7 (Figure 4.4b) for the Movers(2) sample. Similar to the results from the test for Markovity in section 4.3, the distinctive feature was the magnitude of the calculated Chi-square values in both cases. The same applies to testing stationarity at chain level, Figure 4.5. The margin of these Chi-square values exceeding the critical value at 5% significance level was so great that it can be deduced conclusively that for the Movers(2) sample, the data did not show seasonal and trend stationarity at both individual state and chain level. In other words, the customers' behaviour in this particular sample was not constant through time, whether within a calendar year or over the years given the same month when one considers all states i collectively or individually. Overall it means that there were transition movements due to seasonal effect and there was a trend in the movements. In Figure 4.4, only results from PROBE state 1 and 7 were presented. It is because N_{1j} dominated in number and N_{7j} represented the bottom end of the delinquency spectrum. The choice is consistent with the P_{ij} series presented in Chapter 3 (i.e. P_{11} and P_{78}).

Figure 4.4(a) PROBE state 1

Seasonal stationarity			
Year	cal. Chi-square	critical value	degrees of freedom
1996	11833	85.9649	66
1997	10891	85.9649	66
1998	20626	85.9649	66
1999	10223	85.9649	66
Trend stationarity			
Month	cal. Chi-square	critical value	degrees of freedom
January	578	28.8693	18
February	6878	28.8693	18
March	1684	28.8693	18
April	626	28.8693	18
May	3072	28.8693	18
June	716	28.8693	18
July	3544	28.8693	18
August	1249	28.8693	18
September	422	28.8693	18
October	4206	28.8693	18
November	788	28.8693	18
December	2051	28.8693	18

Figure 4.4(b) PROBE state 7

Seasonal stationarity			
Year	cal. Chi-square	critical value	degrees of freedom
1996	292	85.9649	66
1997	394	85.9649	66
1998	526	85.9649	66
1999	571	85.9649	66
Trend stationarity			
Month	cal. Chi-square	critical value	degrees of freedom
January	78.62	28.8693	18
February	57.11	28.8693	18
March	143	28.8693	18
April	68.17	28.8693	18
May	86.24	28.8693	18
June	74.95	28.8693	18
July	134	28.8693	18
August	78.07	28.8693	18
September	141	28.8693	18
October	154	28.8693	18
November	44.99	28.8693	18
December	179	28.8693	18

Figure 4.4 Results from testing seasonal and trend stationarity on the Movers(2) sample at state level for PROBE (a) state 1 and (b) state 7

Figure 4.5(a) seasonal stationarity at chain level for all i, j and t

Year	cal. Chi-square	critical value	degrees of freedom
1996	27800	858.582	792
1997	20209	858.582	792
1998	31296	858.582	792
1999	26007	858.582	792

Figure 4.5(b) trend stationarity at chain level for all i, j and t

Month	cal. Chi-square	critical value	degrees of freedom
January	2818	251.286	216
February	10760	251.286	216
March	4303	251.286	216
April	3809	251.286	216
May	5673	251.286	216
June	3236	251.286	216
July	8892	251.286	216
August	5932	251.286	216
September	2953	251.286	216
October	9603	251.286	216
November	2612	251.286	216
December	5771	251.286	216

Figure 4.5 Results from testing (a) seasonal and (b) trend stationarity on the joint hypothesis at chain level on the Movers(2) sample

4.6 Maximum Likelihood Estimation for fitting interest rate

In section 3.8 the exact functional form and the choice of explanatory variables of the forecast model, Equation 3.5, was explained. Each estimated transition probability must be positive to indicate such transition is possible, or equal to zero to indicate such transition is not possible; and the sum of transition probabilities for a given state i must equal to one. A whole new mechanism and procedure were needed to estimate the values of the parameters associated with each explanatory variable. This had not been attempted in this context before, but in this section it will be demonstrated how it was achieved and the results from the derived procedures will be displayed and discussed in the following section.

From section 3.8 and Equation 3.5, what one estimated here was the trend-cycle component, $Tr_{ij}(t,r(t))$, of the transition probability, $P_{ij}(t,r(t))$.

What one has here is ΣN_{ij} experiments of customers in a given state i moving to another state j. The trend-cycle component, Tr_{ij}, of the probability a customer moving to state j from state i is, from Equation 3.5, $Tr_{ij}(t,r(t)) = [a_{ij} + b_{ij}t + c_{ij}r(t)]$. The chance of N_{ij} customers making such transition is $[Tr_{ij}]^{Nij}$. Thus for all j in one time period the probability is $\Pi_j [Tr_{ij}]^{Nij}$, and for over all time periods this probability is $\Pi_t \Pi_j [Tr_{ij}]^{Nij}$. If one wishes to calculate the maximum likelihood estimation (MLE), the parameters a_{ij}, b_{ij}, and c_{ij} must be chosen to maximise this expression. There are restrictions to be met since this is a set of transition probabilities for a given state i that requires each $Tr_{ij} \geq 0$ and $\Sigma_j Tr_{ij} = 1$. So mathematically to find the MLE for a_{ij}, b_{ij} and c_{ij} over all j and t for a given i, one needs to solve the following optimisation problem:

$$\mathbf{MAX}\ \Pi_t \Pi_j [a_{ij} + b_{ij}t + c_{ij}r(t)]^{Nij} \qquad \textbf{Equation 4.1}$$

subject to

$\Sigma_j [a_{ij} + b_{ij}t + c_{ij}r(t)] = 1$, and $[a_{ij} + b_{ij}t + c_{ij}r(t)] \geq 0$ for all i, j, and t.

Taking a logarithmic transformation and maximising Equation 4.1 results in a non linear optimisation programme:

$$\mathbf{MAX}\ \Sigma_t \Sigma_j N_{ij}(t) \log[a_{ij} + b_{ij}t + c_{ij}r(t)] \qquad \textbf{Equation 4.2}$$

subject to constraints

1. $\sum_j a_{ij} = 1$, $\sum_j b_{ij} = 0$, $\sum_j c_{ij} = 0$ for all t;

2. $a_{ij} + MIN[b_{ij}*T,0] + MIN[c_{ij}*R,0] \geq 0$;

3. $a_{ij} + MAX[b_{ij}*T,0] + MAX[c_{ij}*R,0] \leq 1$;

where T is the maximum value of t and R is the maximum value of r(t). These constraints together guarantee the conditions listed in Equation 4.1 hold, and the estimated $Tr_{ij}(t,r(t))$ to fall in the expected range of 0 and 1. They guarantee these conditions hold at least within T and R, i.e. within the period which the model was fitted.

Proof:

If $t = 0$, $r(t) = 0 \Rightarrow \sum_j a_{ij} = 1$; if $t = 1$, $r(t) = 0 \Rightarrow \sum_j [a_{ij} + b_{ij}] = 1 \Rightarrow \sum_j b_{ij} = 0$;

if $t = 0$, $r(t) = 1 \Rightarrow \sum_j [a_{ij} + c_{ij}] = 1 \Rightarrow \sum_j c_{ij} = 0$

$\therefore$ constraint 1

$0 \leq MIN_{t,r(t)} [a_{ij} + b_{ij}t + c_{ij}r(t)] = a_{ij} + MIN_t\, b_{ij}t + MIN_{r(t)}\, c_{ij}r(t)$,

since $[a_{ij} + b_{ij}t + c_{ij}r(t)] \geq 0\ \forall\ t, r(t) \Rightarrow a_{ij} + MIN[b_{ij}*T,0] + MIN[c_{ij}*R,0] \geq 0$

$\therefore$ constraint 2

$1 \geq MAX_{t,r(t)} [a_{ij} + b_{ij}t + c_{ij}r(t)] = a_{ij} + MAX_t\, b_{ij}t + MAX_{r(t)}\, c_{ij}r(t)$,

since $[a_{ij} + b_{ij}t + c_{ij}r(t)] \leq 1\ \forall\ t, r(t) \Rightarrow a_{ij} + MAX[b_{ij}*T,0] + MAX[c_{ij}*R,0] \leq 1$

$\therefore$ constraint 3

Thus Equation 4.2 describes the maximum logarithmic likelihood for a customer in a given state i to make a transition to state j at time t. The $[a_{ij} + b_{ij}t + c_{ij}r(t)]$ part of Equation 4.2 describes the $Tr_{ij}(t,r(t))$ component of the probability of such transition as a function of the chosen explanatory variables. The b_{ij} parameter describes the contribution of the variable t to the estimated $Tr_{ij}(t,r(t))$. The c_{ij} parameter describes the contribution of the variable r(t) to the estimated $Tr_{ij}(t,r(t))$. And the a_{ij} parameter is the base value to the estimated $Tr_{ij}(t,r(t))$, that is, the $Tr_{ij}(t,r(t))$ value without the influence of t and r(t). The conditions associated with Equation 4.1 consist of that the sum of $Tr_{ij}(t,r(t))$ for a given i must be one, and each estimated $Tr_{ij}(t,r(t))$ must be equal or greater than zero. The base value of $Tr_{ij}(t,r(t))$ (a_{ij}) must be greater or equal to zero as $Tr_{ij}(t,r(t))$ itself, this follows that sum of all a_{ij} must sum up to one for a given i. As a result the sum of b_{ij} and c_{ij} must sum up to zero for a given i (Equation 4.2, constraint 1). To prove that these constants guarantee $\sum_j [a_{ij} + b_{ij}t + c_{ij}r(t)] = 1$, and $[a_{ij} + b_{ij}t + c_{ij}r(t)] \geq 0$ it is sufficient to note that they ensure the function $[a_{ij} + b_{ij}t + c_{ij}r(t)]$ lies between 0 and 1 at the four vertices (0,0), (0,R), (T,R), (T,0) of the (r(t),t) axes. The linearity of the function then implies it is between 0 and 1 in this convex region, since the maximum and minimum of a linear function over a convex region must be at the vertices (Equation 4.2, constraint 2 and 3).

In order to obtain the optimal values of a_{ij}, b_{ij} and c_{ij} parameters, the following procedures were devised and used to solve Equation 4.2. The SAS statistical software was used. The particular procedure used was PROC NLP. A Quasi-Newton method in sequence was specified to solve a nested problem. No one had attempted fitting a function into a Markov Chain before. One was concerned with the compliance of the constraints. Default options had been set. There are numerous options associated with the procedure including one which gives the approximate standard errors associated with the estimated parameters. This allows one to assess the significance of the estimated parameters. In the subsequent results in Figure 4.6, 4.7 these errors are not reported for reason of space, but they were all an order of magnitude less than the estimates themselves.

Initial values of each of the a_{ij}, b_{ij} and c_{ij} parameters were taken to be zero. MAX $\Sigma_t \Sigma_j N_{ij}(t) \log[a_{ij} + b_{ij}t]$ was solved first, then the optimal a_{ij} and b_{ij} from solving this was fed as the initial values to solve MAX $\Sigma_t \Sigma_j N_{ij}(t) \log[a_{ij} + b_{ij}t + c_{ij}r(t)]$. A global optimum solution was not achieved in just one single round of optimising. So the solutions from each round of optimisation was fed into the next round until no further improvement was possible in the objective function value. To ensure the $Tr_{ij}(t,r(t)) \geq 0$ condition (Equation 4.1) must be satisfied, a penalty of -10^{-9} (since $\log(0) = \infty$) was assigned to the objective function value to force the objective function out of local optima and to yield a better objective function value, if not the global optimum. Only non-zero observed entries of N_{ij} were used in the optimisation programme.

4.7 Results for fitting interest rate

In the following sub sections, the results from the maximum likelihood estimation (MLE) on $Tr_{ij}(t,r(t))$ will be presented and interpreted. The MLE was performed to both Movers(1) and Movers(2) samples on the fine state space (Figure 3.5). Recalling from Chapter 3, Movers(2) was a homogenous sample which consisted of customers with full sets of account history in the study period. This was also why tests for stationarity and Markovity were only performed on the Movers(2) sample (section 4.2 and 4.4). On the other hand, Movers(1) described behaviour into account closures but otherwise was a set of maturing accounts. The fine state space (Figure 3.5) was used because a reduced state space (Figure 3.6) would generalise results.

4.7.1 Estimated parameters

The optimal estimated values of parameters $a_{ij}(t,r(t))$, $b_{ij}(t,r(t))$ and $c_{ij}(t,r(t))$ for the Movers(1) and Movers(2) sample produced from the procedures described in section 4.6 were displayed in matrix form, **Q** and **R**, and **R** (section 3.6), in Figure 4.6 and

4.7 respectively. An absolute zero entry in these matrices indicates that particular transition was physically impossible. A minute entry (e.g. 10^{-5}) indicates that particular transition was sparsely frequent. The "total" column at the right hand side of each matrix in Figure 4.6 and 4.7 confirmed that the constraints associated with Equation 4.2 had been satisfied.

Figure 4.6(a) optimal $a_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5	6
0	0.75302	0.12538	0.024705	0.0003136	-4.99E-06	0.0066465	0.0009412
1	0.0098	0.90244	0.065628	0.009675	1.74E-05	0.0092952	0.0031482
2	0.01085	0.25511	0.612264	0.0335045	0.004116	0.0450446	0.0208092
3	0	0.49707	0.118412	0.1722928	0.0009159	0.0378937	0.143732
4	0.02465	0.14085	0.074472	0.0221501	0.7110805	0.0048076	0.01125
5	0.00534	0.17577	0.090666	0.0122148	4.91E-05	0.6991338	0.0120399
6	0	0.1932	0.182095	0.0422377	0.0009249	0.1172605	0.1309867
7	0	0.10749	0.175617	0.0281409	-2.27E-05	0.1024463	3.52E-20
8	0	0.08198	0.098014	0.0023561	0.0004379	0.0493818	0.0185408
9	0.00751	0.01553	0.0051	0.0001356	-1.63E-05	8.96E-09	0.0015693

i \ j	7	8	9	10	11	12	total
0	0	0	2.33E-05	0.088971494	0	0	1
1	0	0	2.37E-05	-0.000033167	0	0	1
2	0.004128249	0.001308313	6.25E-05	0.01280117	0	0	1
3	0.016687429	0	0.00014533	0.012854474	0	0	1
4	0	0	9.45E-06	0.010731504	0	0	1
5	0	0	0.000233574	0.004554861	0	0	1
6	0.271459514	0.018862048	0.000572304	0.042397897	0	0	1
7	0.037814674	0.423134037	0.001956141	0	0.123425	0	1
8	0.004617629	0.629375241	0.005798343	0	0.10950263	0	1
9	0.00020478	-0.000896683	0.939668292	0	0	0.0311963	1

The resultant optimal $a_{ij}(t,r(t))$ parameter matrix for Movers(1) and Movers(2) sample largely agree with each other despite some tiny differences in value, Figure

4.6(a) and Figure 4.7(a). These were the base values of $Tr_{ij}(t,r(t))$ with $t = 0$ and $r = 0$.

Figure 4.6(b) optimal $b_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5	6
0	-0.0009	0.00058	0.000142	-4.52E-06	5.00E-06	2.92E-05	9.89E-06
1	-0.0002	-0.001	0.001159	1.54E-05	-5.81E-07	-3.05E-05	1.14E-05
2	-3E-05	-0.0002	0.000628	-2.07E-05	2.85E-05	-0.000179	-3.62E-05
3	0	0.00175	0.000232	-0.001263	9.81E-07	-0.000861	7.64E-05
4	0.00014	0.00059	-0.002473	-0.000489	0.0029883	-1.95E-05	-0.000375
5	-0.0002	0.00044	0.001599	5.01E-05	1.64E-06	-0.001735	-2.97E-05
6	0	0.00051	0.00054	-0.00012	1.20E-05	-0.000396	0.0001509
7	0	0.00068	0.000877	0.0002176	2.27E-05	0.0002055	-3.45E-20
8	0	0.00055	0.000308	-7.85E-05	-9.79E-06	0.0001893	2.19E-05
9	0.00015	0.00038	9.91E-05	-4.52E-06	1.63E-05	-2.99E-10	4.66E-06

i \ j	7	8	9	10	11	12	total
0	0	0	-7.77E-07	0.0001883	0	0	-3E-09
1	0	0	-7.89E-07	3.32E-05	0	0	8E-10
2	-8.62E-06	1.54E-05	-2.08E-06	-0.000207	0	0	5.6E-09
3	0.0001165	0	-4.84E-06	-4.71E-05	0	0	4.3E-09
4	0	0	1.25E-06	-0.000358	0	0	-2E-09
5	0	0	-7.79E-06	-0.000152	0	0	-6E-09
6	-0.000766	0.0002	-1.91E-05	-0.000117	0	0	1.1E-09
7	9.24E-05	-0.001494	-6.52E-05	0	-0.00054	0	-1E-09
8	3.74E-05	-0.001546	-0.000193	0	0.000724	0	-3E-09
9	1.03E-05	0.000897	-0.002889	0	0	0.001339	1.7E-09

The resultant optimal $b_{ij}(t,r(t))$ parameter matrix for the Movers(1) and Movers(2) sample, Figure 4.6(b) and Figure 4.7(b), described the effect of the variable t on $Tr_{ij}(t,r(t))$. One striking feature difference is apparent. The variable t had opposite effects on $Tr_{i1}(t,r(t))$ for the two samples. While for the Movers(1) sample, t had a positive effect on $Tr_{i1}(t,r(t))$; t had a negative effect on $Tr_{i1}(t,r(t))$ for the Movers(2)

Figure 4.6(c) optimal $c_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5	6
0	0.02193	-0.0093	-0.002788	2.75E-05	5.74E-05	-0.000786	-0.000112
1	0.00077	0.00081	-0.001553	-0.000373	7.18E-06	-0.000683	-0.00018
2	-0.001	-0.0005	0.005698	-0.000704	-0.000197	-0.00256	-0.000353
3	0	-0.0376	0.025812	0.0059492	-0.000114	0.0046221	0.004279
4	-0.0015	-0.0127	0.017188	3.28E-06	-0.008907	-0.000277	0.0013419
5	0.00104	-0.0003	0.001311	0.0017844	-6.54E-06	-0.005179	0.0002638
6	0	-0.0101	0.004498	-0.00039	-0.000123	-0.004012	0.0154576
7	0	-0.0071	-0.006764	-0.002863	9.55E-05	-0.003292	0.0089493
8	0	-0.0081	-0.006213	0.0006305	8.16E-05	-0.003205	0.0004915
9	-0.001	-0.0016	-0.00068	4.06E-05	2.17E-06	3.70E-05	-0.000209

i \ j	7	8	9	10	11	12	total
0	0	0	2.66E-06	-0.009024	0	0	4E-10
1	0	0	4.67E-07	0.0012064	0	0	0
2	-0.00022	3.47E-05	1.08E-06	-0.000284	0	0	3.2E-09
3	-0.002225	0	1.63E-05	-0.00075	0	0	-9E-10
4	0	0	7.54E-05	0.0047671	0	0	1.2E-09
5	0	0	1.11E-05	0.0010478	0	0	5.2E-09
6	0.0010291	-0.002515	3.29E-05	-0.003838	0	0	-1E-09
7	0.0030151	0.018488	6.94E-05	0	-0.01056	0	-4E-10
8	-0.000406	0.025253	0.0001658	0	-0.00872	0	-7E-10
9	-2.73E-05	0.000368	0.0043302	0	0	-0.00129	-4E-09

Figure 4.6 Optimal estimated parameters for Movers(1) sample

sample. This is because Movers(1) matured over time with improving quality. And Movers(2) was homogeneous in time with deteriorating quality.

The resultant optimal $c_{ij}(t,r(t))$ parameter matrix for the Movers(1) and Movers(2) sample, Figure 4.6(c) and Figure 4.7(c), described the effect of the variable r(t) on $Tr_{ij}(t,r(t))$. Like the variable t, variable r(t) had opposite effects on $Tr_{i1}(t,r(t))$ for the Movers(1) and (2) samples.

For the Movers(1) sample, r(t) had a negative effect on $Tr_{i1}(t,r(t))$; and had a positive effect on $Tr_{i1}(t,r(t))$ for the Movers(2) sample. This suggests Movers(2) were more sensitive to changes in r(t) (i.e. interest rates). This is because bad customers in Movers(1) had a exit route, those in the Movers(2) had not (the distinction is explained in section 3.2). While those in Movers(2) did not necessarily return from delinquency to being current in one move, one can assume a borrower did not wish to suffer further interest charges in rising interest rates.

Figure 4.7(a) optimal $a_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5
0	0.796582	0.18198	0.0149	0.000756	0.000815	0.004389
1	0.011482	0.93831	0.03642	0.004584	9.27E-05	0.008196
2	0.004975	0.23743	0.6873	0.022085	0.003092	0.032016
3	-2.20E-07	0.32275	0.22511	0.24232	0.000384	0.085315
4	0.012177	0.08311	0.16727	0.022412	0.685696	0.004829
5	0.003969	0.17507	0.01634	0.009379	1.09E-05	0.789951
6	4.81E-05	0.14975	0.21894	0.057012	0.000141	0.117964
7	0.000164	0.08241	0.13953	0.017234	-8.65E-06	0.072949
8	-3.93E-07	0.03968	0.06253	0.013697	0.000584	0.031973
9	0.00507	0.03342	0.0033	0.001779	-2.40E-06	0.000473

i \ j	6	7	8	9	total
0	0.000555	0	0	1.89E-05	1
1	0.000903	0	0	1.53E-05	1
2	0.010558	0.001267	0.00093	0.0003549	1
3	0.120622	0.0021	0	0.0014001	1
4	0.022856	0	0	0.0016495	1
5	0.004917	0	0	0.0003625	1
6	0.142794	0.297093	0.01183	0.0044331	1
7	-2.8E-05	0.056372	0.59728	0.0340976	1
8	0.019026	0.002382	0.73869	0.0914491	1
9	0.001137	0.00019	0.00998	0.9446597	1

Figure 4.7(b) optimal $b_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5
0	0.002284	-0.0021	-0.0001	-8.96E-06	8.40E-06	-6.4E-05
1	-6.5E-05	-0.0006	0.0007	-4.5E-05	6.98E-07	-3.1E-05
2	-4.9E-05	-0.0011	0.00155	-0.00018	4.27E-05	-0.00019
3	2.20E-07	-0.0014	0.00196	-0.00102	-4.23E-06	4.38E-05
4	-0.00019	-0.0014	-0.0018	-0.0003	0.00405	-6.5E-05
5	-7.8E-05	-0.0002	0.0015	0.000114	1.02E-06	-0.00142
6	4.37E-17	-0.0012	0.00102	-0.00026	-8.97E-07	-0.00017
7	5.71E-11	-0.0012	2.46E-05	-0.00019	8.65E-06	-0.00066
8	3.93E-07	-0.0004	0.00014	-8.5E-05	6.56E-06	-0.00039
9	-0.00011	-0.0007	-7E-05	-3.7E-05	2.40E-06	-9.86E-06

i \ j	6	7	8	9	total
0	-1.49E-06	0	0	-2.19E-07	3.6E-19
1	-1.1E-05	0	0	-1.23E-07	1E-09
2	-6E-05	7.30E-06	1.20E-05	1.10E-07	2.1E-09
3	0.000429	3.58E-06	0	9.85E-07	5.4E-09
4	-0.00028	0	0	8.95E-06	6E-09
5	7.41E-05	0	0	-2.93E-07	3E-09
6	-0.00064	0.0012	9.15E-05	5.55E-06	5.2E-09
7	2.77E-05	-0.00036	0.00233	2.57E-05	-3E-09
8	-0.0004	-3.3E-05	0.00105	0.0001412	-1E-09
9	-2.4E-05	-3.97E-06	-0.0002	0.0011506	-6E-09

Figure 4.7(c) optimal $c_{ij}(t,r(t))$-parameter matrix

i \ j	0	1	2	3	4	5
0	0.005998	-0.0052	-0.0006	-2.6E-05	-6.3E-05	-0.00013
1	6.52E-06	-0.0042	0.00416	0.000448	-8.84E-06	-0.00057
2	-0.00018	0.0069	-0.0077	0.001356	-0.00024	-0.00076
3	2.93E-08	-0.0017	0.00848	-0.00666	-2.4E-05	-0.00479
4	0.001821	0.00473	0.00392	-0.001	-0.00823	4.77E-05
5	0.000763	0.00259	0.01248	0.001818	-1.45E-06	-0.01818
6	-6.41E-06	0.00355	0.00124	-0.00176	3.20E-05	-0.00395
7	-2.2E-05	0.00492	0.00564	0.000381	0.000112	0.006365
8	5.23E-08	0.00237	0.00137	-0.00075	2.75E-05	0.002282
9	0.000117	0.00027	5.5E-05	1.23E-05	2.26E-06	3.73E-05

i \ j	6	7	8	9	total
0	-4.1E-05	0	0	-1.11E-06	5.7E-19
1	0.000157	0	0	-1.24E-06	6E-09
2	0.000765	-1.6E-05	-6E-05	-4.73E-05	1E-09
3	0.004772	0.00011	0	-0.000187	3E-10
4	-0.00107	0	0	-0.00022	3E-10
5	0.000577	0	0	-4.46E-05	-2E-09
6	0.018621	-0.01567	-0.0015	-0.000591	-2E-09
7	0.01165	0.002401	-0.0269	-0.004546	1E-09
8	0.002861	0.000292	0.00374	-0.012193	5E-10
9	-1.96E-19	1.48E-05	0.00013	-0.000645	-1E-09

Figure 4.7 Optimal estimated parameters for Movers(2) sample

4.7.2 Estimated $P_{ij}(t,r(t))$

In this section the accuracy of the forecast model will be discussed based on the estimated transition probabilities, $P_{ij}(t,r(t)) = Tr_{ij}(t,r(t)) + Sn_{ij}$ (Equation 3.5), in the test period on the Movers(1) and (2) samples. Recalling from section 3.4 where the holdout methods used in this thesis were put forward. The holdout method applied on the Movers(1) sample was the traditional way ("in-sample", test sample collected at the same time as development sample) to evaluate the ability of the forecast model to provide reliable forecasts. The method applied on the Movers(2) sample ("out-of-sample", test sample collected after the development sample), however, tested the robustness of the forecast model over time as well as accuracy. As before the P_{11} and P_{78} series are selected for illustration here, because N_{11} dominated in number and P_{78} represents the opposite end of the delinquency spectrum to P_{11}.

t	Est. $Tr_{11}(t,r(t))$	Actual P_{11}	% error	Est. $Tr_{78}(t,r(t))$	Actual P_{78}	% error
31	0.87642	0.80157	9.33	0.51547	0.49956	3.18
32	0.87538	0.85414	2.47	0.51397	0.56054	-8.31
33	0.87435	0.88532	-1.24	0.51248	0.52546	-2.47
34	0.87311	0.8649	0.95	0.50636	0.57441	-11.85
35	0.87167	0.8547	1.99	0.49562	0.58081	-14.67

Figure 4.8 Estimated and actual P_{11} and P_{78} for Movers(1) sample (not seasonally adjusted)

Figure 4.8 displays the estimated and actual P_{11} and P_{78} transition probabilities of the Movers(1) sample in the test period. Though not seasonally adjusted as time series decomposition not performed on the Movers(1) sample for the reason put forward in section 3.2.3, that some accounts in this particular sample dropped out of the system hence did not have a full set performance records during the study and test period, one can see the magnitude of percentage errors are acceptable.

Figure 4.9 displays the estimated and actual P_{11} and P_{78} transition probabilities of the Movers(2) sample in the test period. On this occasion the estimated transition probabilities were seasonally adjusted. The magnitude of percentage errors are satisfactory. The holdout sample outperformed the estimation in the case of P_{11}. The seasonal adjustment for the month November was particularly strong for both P_{11} and P_{78} of the Movers(2) sample (Figure 3.8, 3.9), which might contributed to the high forecast errors.

t	Est. $P_{11}(t,r(t))$	Actual P_{11}	% error	Est. $P_{78}(t,r(t))$	Actual P_{78}	% error
49	0.90250	0.91078	-0.91	0.51693	0.49931	3.53
50	0.88086	0.91583	-3.82	0.58406	0.54678	6.82
51	0.88359	0.9158	-3.52	0.52152	0.52104	0.09
52	0.893267	0.88275	1.19	0.55383	0.60806	-8.92
53	0.87188	0.91425	-4.63	0.61348	0.58609	4.67
54	0.88797	0.90021	-1.36	0.54342	0.63278	-14.12
55	0.88957	0.90528	-1.74	0.52299	0.55189	5.24
56	0.87313	sys. error	-------	0.63494	sys. error	-------
57	0.88926	sys. error	------	0.56755	sys. error	-------
58	0.88752	0.91429	-2.93	0.53369	0.56401	-5.38
59	0.85398	0.90893	-6.05	0.62003	0.49908	24.23
60	0.88583	0.88941	-0.40	0.55448	0.56348	-1.60

Figure 4.9 Estimated and actual P_{11} and P_{78} for Movers(2) sample (seasonally adjusted, "sys. error" – system error)

While extracting the test set for the Movers(2) sample, it was discovered that the field that records the account performance of the month September 2000 was not populated. This rendered evaluation of the forecast model for the sample was not possible when t = 56, 57, Figure 4.9. This system error (Figure 4.9) was reported to the data provider.

4.7.3 Interpretation of results – the Good, the Bad and the Ugly

There are two ways one can interpret the results of estimated $Tr_{ij}(t,r(t))$. One is the overall effect of t and r(t) on $Tr_{ij}(t,r(t))$. The other is the estimated value of $Tr_{ij}(t,r(t))$ itself.

From section 4.7.1, it is found that time had a positive effect on $Tr_{i1}(t,r(t))$ for the Movers(1) sample. Namely, customers in this sample were increasingly likely to return to being current after falling delinquent as time went on. Time on the other hand had a negative effect on $Tr_{i1}(t,r(t))$ for the Movers(2) sample. It is because Movers(2) was homogeneous in time with deteriorating quality.

Base rate had a negative effect on $Tr_{i1}(t,r(t))$ for the Movers(1) sample except when i = 1. But it had a positive effect on $Tr_{i1}(t,r(t))$ for the Movers(2) sample except when i = 0, 1, 3. Movers(1) would fall victim of an interest rate rise unless the customer was already being current. But Movers(2) was sensitive to an interest rate rise, they were not prepared to suffer high interest payments. The exceptions can be explained as state 0 and 1 (in current) being the starting point for most cases were insensitive to interest rate changes. And state 3 was only an infant stage of delinquency, customers in this state were probably unaware of their financial position until they received their monthly statement.

$Tr_{i1}(t,r(t))$ was illustrated as an example here. The effect of time and interest rate changes can be interpreted in similar fashion for other individual $Tr_{ij}(t,r(t))$ series.

A full list of the estimated $Tr_{ij}(t,r(t))$ for the Movers(1) and Movers(2) samples at selected t were listed in Appendix 9.2.1 and 9.2.2 respectively. A full list of the actual P_{ij} at identical t were listed in Appendix 9.3.1 and 9.3.2 respectively for Movers(1) and Movers(2). The particular t were carefully selected to fairly reflect the estimated $Tr_{ij}(t,r(t))$ at different times of a year at different years. Equation 4.2 guarantees the condition that the estimated $Tr_{ij}(t,r(t))$ will fall between 0 and 1 in the study period. But one did experience few $Tr_{ij}(t,r(t))$ anomalies in the forecast period for the Movers(2) sample. It was due to the holdout method used to this sample (holdout set was collected after the model was fitted). The magnitude of T was much bigger than R (T = 60, R = 7.5). When b_{ij} was negative and c_{ij} was positive, $c_{ij}R$ was not enough to compensate $b_{ij}T$. As a result estimated $Tr_{ij}(t,r(t))$ fell below 0. Similarly when b_{ij} was positive and c_{ij} was negative, estimated $Tr_{ij}(t,r(t))$ would fall outside 1. To remedy this one can raise T to the end of the forecast period (e.g. T = 60, in this case), and raise R to an educated estimation.

Overall, the estimated $Tr_{ij}(t,r(t))$ for the Movers(1) and Movers(2) sample largely agreed with each other despite the difference between the two samples. The terminating behaviour, $Tr_{i10}(t,r(t))$, $Tr_{i11}(t,r(t))$ and $Tr_{i12}(t,r(t))$, were not enough to cause significant attention. This might be due to the nature of the product this thesis concentrated efforts on, i.e. personal current account. Terminating behaviour would probably be causing more concerns in other revolving credit products.

Common to both Movers(1) and Movers(2) samples, there existed a critical delinquency state where the likelihood of recovering from debt, falling deeper into delinquency and staying put in the next coming month was equally probable. This state is PROBE state 6, when customer is in debt for over a month. Once customers fell into state 7 his/her chances of falling further behind doubled (from $Tr_{67}(t,r(t))$ to $Tr_{78}(t,r(t))$).

To summarise, there were three distinctive types of customers in the Jan1996-Live sample (Figure 3.1, section 3.2). The Good, were those who were able to return to being current after falling into debt and sensitive to interest rate changes. The Bad, were those who fell delinquent and not bad enough to be expelled but found it difficult to return to being current. The Ugly, obviously were those who had closed bad or had been written off. The terms the Good, the Bad and the Ugly do not reflect and bear no relation to any official descriptions of customers.

4.8 Conclusion

In this chapter the tests for stationarity and Markovity used were laid out and explained. The test results rebutted the assumptions taken by previous authors, namely that transition probabilities were not constant over time and state transitions did not follow a First Order Markov Chain. Mechanisms and procedures to estimate the model parameters were formulated. Constraints were imposed to ensure certain conditions must be complied, namely that the estimated transition probabilities must take a value in the range of 0 and 1; and that the sum of estimated transition probabilities for a given state i must equal to one.

The estimated parameters had different impacts on the estimated transition probabilities but the combined effect was linear. There was a slight imperfection in the forecast model. A remedy was offered. Otherwise it has been an accurate and robust model.

We concluded that there was a critical delinquency state in the Markov Chain. And we saw three types of customer behaviour, which we called Good, Bad and Ugly. This idea of segmenting the population will be pursued in the next chapter in an effort to restore the Markovity of the chain models.

5. Segmentation

Results from the testing for first and second order Markovity on the Movers(2) sample in section 4.3 are far from satisfactory. The calculated Chi-square values conclusively exceeded the critical value by a huge margin. However, the results improved by incorporating more past data into a given state i. This means one is more confident about an account's future transition given his current and past performance history of that particular account. Iterative procedures as described in Weiss et al (1982) were initiated to seek for a more suitable state-space, segmentation or both in order to improve the predictive power of the resultant Markov Chain model. Results from these procedures will be presented in this chapter. Justifications for the state-space and segmentation chosen will be provided and discussed.

5.1 Why segment Movers – need for another state-space

5.1.1 The need for a new state-space

Schniederjans and Loch (1994) found small and zero entries in transition matrices one of the major hindrances in implementing a non-stationary Markov model with real banking data (Chapter 2). They found this was the result of a small sample size. A similar situation was observed even in huge data samples here. Consequently only non zero entries were used in the tests and procedures used in Chapter 4. The problem of zero and small entries is characteristic of transition matrix based modelling when a state-space that represents a scale of relative measure is used, e.g. high-low, good-bad, recovered-deceased, etc.. The state-space as defined by PROBE (Appendix 9.1) is a good example. The problem is more apparent with small samples, but large samples cannot escape such eventuality. Consider the **Q** matrix in section 3.6 (Figure 5.1) which describes the transitional behaviour of accounts which remained open. The majority of accounts would occupy the diagonal entries, which means accounts making transitions between adjacent states. Top right corner of the **Q** matrix is not occupied, i.e. non entries, because these transitions are physically impossible, for example a transition from state 1 to state 8 would require an account

$$P(t) = \begin{array}{c|ccccc|}
 & 0 & 1 & 2 & \cdots & 9 \\ \hline
0 & & & & & \\
1 & & & & & \\
2 & & & & & \\
\vdots & & & & P_{ij}(t) & \\
9 & & & & & \\ \hline
\end{array}$$

Figure 5.1 Q matrix

being in current to become more than 3 months delinquent in a calendar month's time, which is, of course, not possible. The bottom left of the matrix represents those accounts making transitions beyond adjacent states, namely from being delinquent for a considerable length of time to being current in a calendar month. And this is where sparse and zero entries persist. The best a large sample can do in this case was to fill up this part of the matrix. When more past history were considered, even a large sample could not escape zero entries.

As mentioned before the number of transitions expand exponentially with m^{n+1} (where n is the order of the Markov Chain) when one incorporates more past history. For the Movers(2) sample in terms of the state-space as prescribed by PROBE at fine state level, this means the **Q** matrix would grow vertically by ten fold if one is interested in $P_{hij}(t)$. When $P_{ij}(t)$ constitutes 100 possible entries (states 0 to 9), $P_{hij}(t)$ makes up a state-space for 1000 possible entries (10 states by 10 states by 10 states). While it is feasible with today's software and technologies to carry out those tests and procedures described in Chapter 4, it would have been tedious for one to perform such a task. It certainly is not practical nor sensible to do so given the time scale of this thesis. As a result a new state-space was needed to simplify the tasks, but yet retain the measure of goodness of behaviour. A logical way was to "lump" states together, that is, to amalgamate several fine states of similar behaviour together to form a new, broader performance status on a "good" to "bad" scale. Such definitions were already stated in PROBE definitions (Appendix 9.1). The original 13 detailed states were collectively grouped into 3 stati: Good (G), Indeterminate (I), Bad (B). Each status consists of 3 to 5 fine states; Good(G) – 3, Indeterminate(I) – 5, and Bad(B) – 5. If closure states (PROBE states 10, 11 and 12) are omitted then the composition is: Good(G) – 2, Indeterminate(I) – 5, Bad(B) – 3. These compositions show it is progressively harder for a customer to be defined as "good". G = {state 1, 2}, I = {state 0, 3, 4, 5, 6}, B = {state 7, 8, 9}.

These 3 stati formed the new state-space that was used in the analysis which follow in this chapter, Figure 5.2. Since the state space has been reduced a customer can now make a transition from Good to Bad (e.g. from PROBE state 2 to 7) in one calendar month just as from Bad to Good. So the new state space does not have the same non-entry problem at the top right corner of the matrix as the **Q** matrix does.

$$
\begin{array}{cc|ccc|}
 & & t+1 & & \\
 & & j & & \\
 & i & G & I & B \\
\hline
t & G & N_{GG}(t) & N_{GI}(t) & N_{GB}(t) \\
 & I & & & \vdots \\
 & B & & \cdots & N_{BB}(t) \\
\hline
\end{array}
$$

Figure 5.2 New state-space based on Good(G), Indeterminate(I) and Bad(B) status

Unless stated otherwise, this new 3-status state-space was used in the analysis performed throughout this chapter.

5.1.2 Segment the Movers(2) sample

The Markovity test results on the Movers(2) sample (section 4.3) given the reduced state-space showed incorporating more past history made the calculated Chi-square values approach more to the critical value. Incorporating past history was not sufficient to make the sample satisfy the Markov assumption. In addition to a new state-space the Weiss et al (1982) procedure included the search for a set of suitable sub-populations. The same applied here. The reason was two fold. The obvious one was to satisfy the Markov assumption. The second was the fact that segmentation provides better understanding of the behaviour of distinctive groups of customers, since the ultimate use of the final predictive model would be for the purpose of policy analysis. The methodology for splitting is two fold. Firstly the subjective opinion of the experts in the bank suggested that there are four types of customer behaviour, namely:

(1) – those accounts which stayed in the same status throughout;
(2) – those accounts which switched from the usual status to another;
(3) – those accounts which switched from the usual status to another then returned;
(4) – those accounts which made frequent transitions across status.

This idea that customers segment by the number of state switches they make, means one can analyse this statistically by segmenting using this variable. So the second approach is what this thesis attempts in the following sections.

These definitions above were not inclusive of all behaviour but was believed could largely capture the majority of accounts. However, an objective way of defining the split between these proposed sub-populations was clear. What these proposals described was the number of transitions an account made given the new state-space during the study period (i.e. January 1996 to January 2000).

So the Movers(2) sample was split on the number of transitions made during the study period based on the new state-space (i.e. Good/Indeterminate/Bad). The results were as follows, Figure 5.3. This criterion was a subjective one, but as will be demonstrated in the following sections it was an effective measure for studying the dynamic behaviour of bank customers here.

We want to segment Movers (2) to improve the Markovity of the sample. Work in other areas and Frydman et al (1985) on consumer credit suggests that a Mover-Stayer model has its merits. We believe that expanding this model by segmenting on the frequency of movement will improve the Markovity of credit behaviour. We will test this out in this chapter. The interests are then on which application/behavioural characteristics will predict which frequency of movement segment a customer will be in (Chapter 6).

Again the Movers(2) sample was selected because each case in the sample had a full set of account history.

Figure 5.3 shows a little under half of the accounts in the Movers(2) sample made transitions within one status (i.e. 0-Mover). The figure also shows there is a clear-cut in the percentage of "Even-Movers" and the percentage of "Odd-Movers" in the sample. "Even-Movers" were those accounts which made an even number of transitions during the study period given the new state-space. "Odd-Movers" were those which made an odd number of transitions during the study period given the new state-space. The figure shows there were more Even-Movers in the sample than Odd-Movers. For a given Even-Movers group, omitting 0 and >8 transitions, the percentage values of its adjacent Odd-Movers groups were much less than its own. In addition, the percentage values of both Even- and Odd- Movers groups decrease as the number of transitions made increases.

Figure 3.2 in section 3.2.1 shows by far most accounts in the Jan1996-Live sample were in Good status at the beginning of the study period, and one can assume that the same applies to its subsequent subset samples. Given most accounts started off being Good, the large share of Even-Movers suggests that most customers would return to his/her original status (i.e. Good) eventually, if not, he/she would most likely end up Bad. At the other end of the spectrum, the worst an Even-Mover here can do given he/she started off Bad was to stay Bad. If this was not the case an Even-Mover would most likely end up improving his/her status. On the other hand, an Odd-Mover started off Good would most likely end up in Indeterminate or Bad.

Number of transitions made	Frequency	Percentage (%)
0	140045	44.4
1	19222	6.1
2	51617	16.4
3	11800	3.7
4	23482	7.4
5	8496	2.7
6	13790	4.4
7	6458	2
8	8863	2.8
>8	31818	10.1
total	315591	100

Figure 5.3 Splits on Movers(2) sample given the new state-space

What one needs now is some measurements to quantify but most importantly of all give some credit to the propositions just made. Figure 5.4 shows the conditional probabilities of the final destinations at the end of the study period (January 2001, t = 49) given the starting status at the beginning of the study period (January 1996, t = 1), for 2-, 3-, 4-, 5- , 6-, 7- and 8- Movers as defined in Figure 5.3 given the new state-space, Good(G)/Indeterminate(I)/Bad(B). Even-Movers and Odd-Movers were grouped together for comparison.

Figure 5.4(a) – Even-Movers

2-Mover	j(t=49)		
i(t=1)	G	I	B
G	0.9893	0.0002	0.0105
I	0.0261	0.9624	0.0115
B	0.1382	0.4660	0.3958
4-Mover	G	I	B
G	0.9690	0.0093	0.0217
I	0.0999	0.8731	0.0270
B	0.5684	0.3526	0.0789
6-Mover	G	I	B
G	0.9434	0.0201	0.0366
I	0.1461	0.8172	0.0367
B	0.6154	0.2756	0.1090
8-Mover	G	I	B
G	0.9195	0.0342	0.0463
I	0.2016	0.7425	0.0560
B	0.6341	0.2973	0.1585

Figure 5.4(b) Odd-Movers

3-Mover	j(t=49)		
i(t=1)	G	I	B
G	0.1426	0.8350	0.0224
I	0.9740	0.0079	0.0181
B	0.8170	0.0766	0.1064
5-Mover	G	I	B
G	0.2262	0.7385	0.0353
I	0.9639	0.0118	0.0244
B	0.7010	0.1649	0.1340
7-Mover	G	I	B
G	0.2724	0.6775	0.0501
I	0.9363	0.0214	0.0424
B	0.7293	0.1934	0.0773

Figure 5.4 Conditional probabilities, P[j(t=49)]|i(t=1)], given the new state-space for (a) Even-Movers and (b) Odd-Movers

Figure 5.4 without doubts validates what was just discussed earlier, note how the diagonal entries, except Bad, for Even-Movers dominated the matrix. The tendency of sliding down the spectrum for Even-Movers strengthened as the number of transitions made increased. However, the tendency of moving away from being Good for Odd-Movers weakened as the number of transitions made increased. This suggests a population homogeneity similar to that discussed in section 3.8.3, Figure 3.11, that there seemed in a long term customers in this particular portfolio shift to the middle ground of the performance spectrum.

5.2 Segmentation Markovity measure

Having decided on the method of splitting the Movers(2) sample (Weiss et al (1982) used formal testing), one now needs to test the Markov assumption again for each of the sub-populations (section 4.2). Figure 4.2 in section 4.3 showed the Chi-square result values converged to critical value when more past history was considered. In theory one can consider incorporating all the past, in this case account performance, history into the test until critical value is achieved. But this makes the state space too big. Given the time of this thesis, it is not practical nor sensible to do so. What was needed here was a quantity that can measure the marginal improvement incorporating one extra past account performance data point would bring. This quantity would also aid one to compare relatively between sub-populations in order to decide whether sub-populations should be either combined or further subdivided, and to decide when optimal convergence has been achieved.

This quantity, "ω", was the "total weighted sum of the likelihood of the Markovity hypothesis being satisfied at the calculated Chi-square values", and is defined by:

$$\boldsymbol{\omega = \sum_i \omega_i = \sum_\sigma \sum_{h,j} N_i * P_{\chi 2(i,\sigma)}} \qquad \textbf{Equation 5.1}$$

for a given individual or combinations of segments(σ), Figure 5.5. The explanation for this measure is as follows.

$P_{\chi 2(i,\sigma)}$ is the probability using the Chi-square test that the transitions from state i, for the population in segment σ appear to satisfy the Markovity assumption. ω is the weighted sum of these probabilities. It is summed over each sub-population in the segmentation and the probabilities are weighted by the total number of accounts in that sub-population. Since the overall total number of accounts is constant, this is like weighting the sub-population by how likely a random case is to be in that sub-population. Thus ω gives the relative ranking of the individual or combination of segments in terms of the likelihood of the Markov hypothesis being true.

f(hi)	f(ij) 11	12	...	1m	total	cal. χ^2
11	N_{111}	N_{112}	...	N_{11m}		
21	N_{211}	...				
⋮	⋮			⋮		
m1	N_{m11}			N_{m1m}		
					N_{h1j}	χ^2
						$P_{\chi2}$
						$\omega = N_{h1j} * P_{\chi2}$

Figure 5.5 How to calculate "ω" for a given i in segment σ

There were two scenarios to consider. If the total entries to a matrix was small then the magnitude of ω would be small regardless of how probable that Chi-square value was. If the total entries was large then the magnitude of ω would be large if the probability was high and vice versa. So the criterion was whichever single or combination(s) of the segment(s) that would produce the largest value of ω would be the optimal segmentation scheme(s) that satisfy (or converge to, at least) the Markov assumption. This was equivalent to saying the matrix for a given i that produced a satisfactory ω value contained the most favourable and probable transitions customers had made during the study period.

5.3 Results of the Markovity measure – choice of segments

Omitting 0-Movers; the test for Markovity as described in section 4.2 was performed on individual and combinations of subsets identified in Figure 5.3 section 5.1.2, for First, Second and higher order Markovity. And the ω value was calculated. Figure 5.3 provided some clues as to how one could create new schemes of Movers by different combinations of individual subsets, based on the share of percentage each subset occupied in the parent Movers(2) sample. As mentioned before, Even-Movers occupied more share than Odd-Movers, so adjacent Even-/Odd- Movers subsets could be combined together to form a new Movers scheme. Another way was simply to combine adjacent subsets together based on the number of transitions made. Transition frequencies over the whole study period were used in the test for Markovity in section 4.2. Given the potential number of subsets or schemes to be tested here, only the first 12 transitions in the study period were used for testing First and Second order Markovity, and the first 8 transitions were used for testing higher order Markovity.

Figure 5.6 summarises the ω results from the Markovity test on the individual and combinations of Mover subsets identified in Figure 5.3. An asterisk in Figure 5.6 next to the value indicates there existed sparse and/or zero entries in the test matrix. These results from Figure 5.6 shows even a large sample size of 4-, 6-, 8- Movers subsets on a reduced state-space cannot escape sparse or zero entries. These results also suggested that Even-Movers only make certain transitions, as subsets like ≥5-Mover and ≥5less6&8-Mover did not suffer on identical test. A "n/a" in Figure 5.6 indicates performing such a test would not yield satisfactory results, so was not carried out.

The ω values from Figure 5.6(a) confirmed that incorporating more past history will make the samples/subsets converge to Markov behaviour. The magnitude of ω values increases as the order Markovity increases. However in this case second order Markovity was optimal in terms of relative improvement in ω when going from n-th to (n+1)-th order Markovity. Given the time scale of this thesis and the occurence of sparse/zero entries in testing for third order Markovity, it would not be productive to carry on testing for higher order Markovity. It was concluded that transitions made by the sub-populations of the Movers(2) sample given the new state-space followed a Second order Markov Chain. That is the immediate future (t+1) performance status of a customer is conditionally dependent on his/her current (t) and immediate past (t-1) performance status.

	ω value (rounded to integer)		
Mover-scheme	First order	Second order	Third order
(a) Individual scheme:			
1-Mover	2340*	2321*	n/a
2-Mover	10^{-24}	209	n/a
3-Mover	10^{-66}	39	n/a
≥3-Mover	n/a	25	n/a
4-Mover	10^{-31}	3335	8961*
≥4-Mover	n/a	6	n/a
≥5-Mover	10^{-241}	16	2576
6-Mover	n/a	626	2213*
8-Mover	n/a	264	2442*
≥5less6-Mover	n/a	17	n/a
≥5less6&8-Mover	n/a	1	2259
(b) Combinations:			
(1+2)-Mover	10^{-12}	803	n/a
(2+3)-Mover	0	79	n/a
(1+3)-Mover	10^{-80}	92	n/a
(2+4)-Mover	10^{-75}	232	n/a
(3+4)-Mover	n/a	180	n/a
(4+6)-Mover	n/a	409	n/a
(6+8)-Mover	n/a	489	n/a
(1+2+3)-Mover	n/a	147	n/a
(2+3+4)-Mover	n/a	119	n/a
(1+2+3+4)-Mover	n/a	155	n/a

Figure 5.6 Summary of ω results from Markovity test on (a) individual or (b) combinations of Mover subsets

What had been discussed so far concerned the individual or individual combination(s) of subset(s)/sub-population(s) of the Movers(2) sample. But none of these subsets or combinations added up to the whole Movers(2) sample. Having decided that these subsets followed a Second order Markov Chain, what was needed was an optimal segmentation that would split the Movers(2) sample into segments which were made up of the subsets already identified, so that these segments too followed a Second order Markov Chain. And eventually a predictive model similar to the one fitted to the Movers(2) sample would be fitted to each of the segments. One could argue since the model fitted to the Movers(2) sample in Chapters 3 and 4 already provided an excellent fit, what could modelling on individual segments additionally do? The answer refers back to the argument put forward in section 5.1.2 that the ultimate use of the resultant predictive models is for policy analysis.

Figure 5.7 summarises the ω results from testing Second order Markovity, the chosen optimal Markov Chain, on different segmentation arrangements on the Movers(2) sample. The results were sorted by ascending number of segments then by ascending value of ω. An asterisk in Figure 5.7 next to the ω value indicates that particular segmentation was the optimum given the number of segments. In terms of ω values alone, scheme 20 was optimal. However, in terms of marginal improvement in ω going from n to (n+1) segments, scheme 10 provided the biggest improvement. As a result it was concluded that scheme 10 was the optimal and chosen segmentation.

To summarise the results so far from section 5.1 to 5.3, a predictive model based on a Second order Markov Chain was to be fitted to each of the segment in a scheme which split the Movers(2) sample into the following subsets based on the number of transitions made during the study period. The identified segments are [1-to-3]-Mover, [4]-Mover, and [≥5]-Mover.

Scheme	Segmentation	No. of segments	ω value (rounded to integer)
1	(1+2+3)-, ≥4-Mover	2	153
2	(1+2+3+4)-, ≥5-Mover	2	171*
3	(1+3)-, (2+4)-, ≥5-Mover	3	340
4	(1+2+3)-, (4+6)-, (≥5less6)-Mover	3	573
5	(1+2+3+4)-, (≥5less6&8)-, (6+8)-Mover	3	645
6	(1+2)-, (3+4)-, ≥5-Mover	3	999
7	1-, (2+3)-, ≥4-Mover	3	2406
8	1-, (2+3+4)-, ≥5-Mover	3	2456
9	1-, 2-, ≥3-Mover	3	2555
10	(1+2+3)-, 4-, ≥5-Mover	3	3798*
11	1-, 2-, (3+4)-, ≥5-Mover	4	2711
12	(1+3)-, 2-, 4-, ≥5-Mover	4	3652
13	(1+2)-, 3-, 4-, ≥5-Mover	4	4192
14	1-, (2+3)-, 4-, ≥5-Mover	4	5751*
15	(1+2)-, (3+4)-, (≥5less6&8)-, 6-, 8-Mover	5	1071
16	(1+3)-, (2+4)-, (≥5less6&8), 6-, 8-Mover	5	1214
17	1-, 2-, 3-, 4-, ≥5-Mover	5	5920*
18	1-, 2-, (3+4)-, (≥5less6&8)-, 6-, 8-Mover	6	3601
19	(1+2), 3-, 4-, (≥5less6&8), 6-, 8-Mover	6	5067*
20	1-, 2-, 3-, 4-, (≥5less6&8), 6-, 8-Mover	7	6794*

Figure 5.7 Summary of ω results from segmentation on Movers(2) sample

5.4 Higher order Markov Chain modelling

The methodology for higher order Markov Chain modelling which was performed was identical to that carried out in section 3.8. The chosen Scheme 10 offered an extra benefit. The percentage share of each subset in the parent Movers(2) sample was fairly comparable: [1-to-3]-Mover – 26.2%, [4]-Mover – 7.4% and [≥5]-Mover – 22% ([0]-Mover – 44.4%). [4]-Mover was understandably dwarfed by the other two because it consisted of only one sub-population where the other two were made up by more than one. Consequently the resultant model fitted would not be affected by sample sizes. [1-to-3]- and [4]- Movers subsets offered little variations in terms of the variety of transitions observed and would serve little interests here, so efforts were concentrated on the [≥5]-Mover subset in the modelling stage. But the final model derived was assumed to be valid to the other subsets and would be fitted accordingly. Therefore the [≥5]-Mover subset was selected as illustration here.

The $P_{hij}(t-1)$ series of the [≥5]-Mover subset was decomposed to its constituent trend-cycle, seasonal and error components to search for the underlying process that generated the series. Then the trend-cycle, Tr_{hij}, component was regressed on explanatory variables t-1 (since status j was proved to be conditionally dependent on status i at time t given status h at time t-1) and av_r (i.e. the average value of base rates r at time t and t-1) to see how time and interest rate affected the component. There were 3^3 (new state-space; performance status at t-1, t and t+1), 27 $P_{hij}(t-1)$ series altogether. Only $P_{GGG}(t-1)$ and $P_{BBB}(t-1)$ series would be illustrated here as representative of the subset, because N_{GGG} dominated in number and N_{BBB} represents the opposite end of the delinquency spectrum.

Figure 5.8 displays the results from time series decomposition on the $P_{GGG}(t-1)$ series. Figure 5.8(a) shows the observed and the smoothed (i.e. trend-cycle component of) $P_{GGG}(t-1)$ series. The first and last 6 data points were omitted from the analysis for the same reason as before. It shows an almost "flat" underlying trend until towards the end of the study period where it shows a slight upward movement. The fluctuations across the underlying trend was not as wild as seen before. Figure 5.8(b) shows the seasonal effect on the $P_{GGG}(t-1)$ series. It shows October was particularly averse to the series. One now has to consider future transition given the status two months previous. Given a customer was being Good at the end of October and November, as the Christmas season approached he/she was likely to find it difficult to remain Good in the coming December.

Figure 5.9 shows the results from time series decomposition on the P_{BBB}(t-1) series, the other end of the delinquency spectrum given the new state-space. Again the trend-cycle component was fairly stable until the latter part of the study period where it surged. And fluctuations across the trend line were less regular. The latter half of a calendar year had a positive influence on customers who were already being Bad in two consecutive months on the chances of him/her remaining Bad in the next month.

Figure 5.8(a)

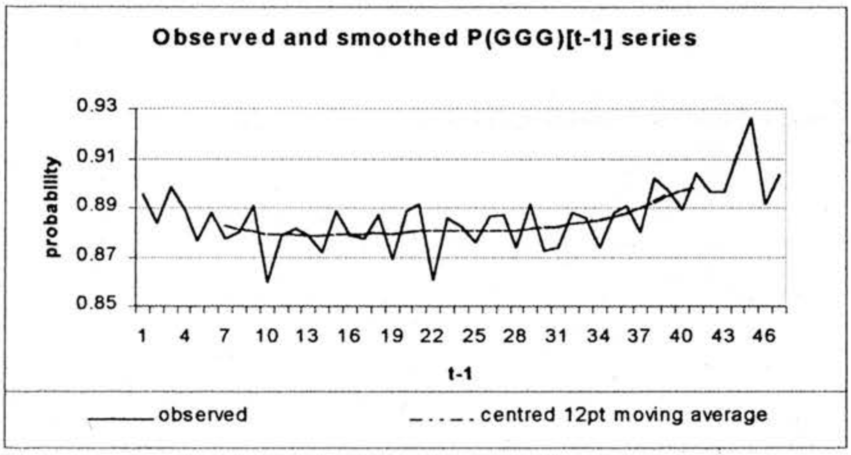


Figure 5.8(b)

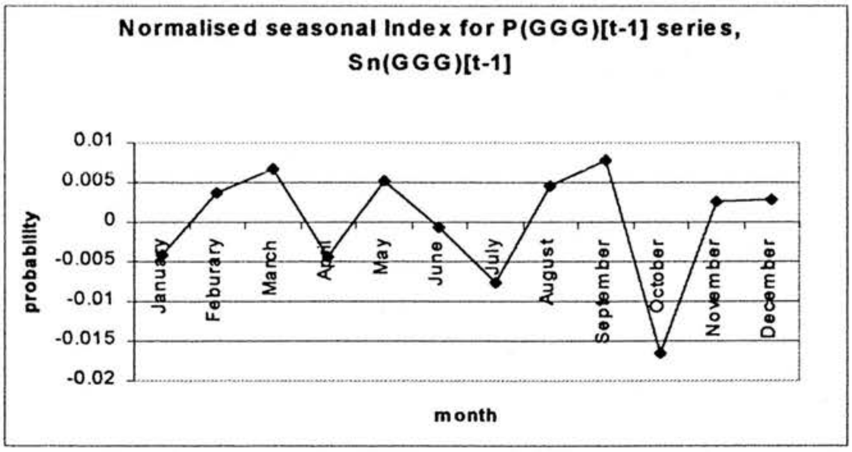


Figure 5.8 Time series decomposition on the P_{GGG}(t-1) series - [≥5]-Mover

Though not displayed here the error components of the two series had irregular runs of positive and negative values.

Figure 5.9(a)

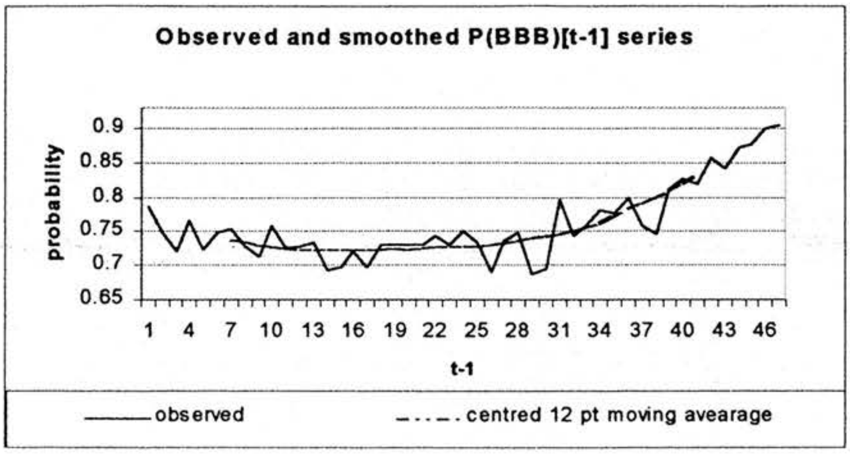


Figure 5.9(b)

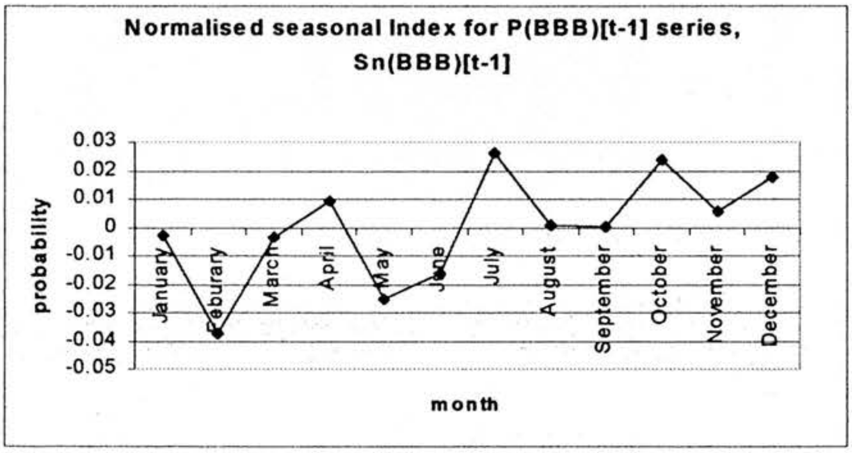


Figure 5.9 Time series decomposition on the P_{BBB}(t-1) series - [≥5]-Mover

5.5 Results of the higher order Markov Chain on segments

Figure 5.8(a) and 5.9(a) showed that the trend-cycle component of the P_{GGG}(t-1) and P_{BBB}(t-1) series were steady through a large part of the study period. A linear regression analysis here might not seem constructive as before. Nevertheless it was performed in order to be consistent with the methodology carried out in section 3.8.

Figure 5.10(a) Regressing $Tr_{GGG}(t-1)$ on t-1

Dependent Variable: $Tr_{GGG}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00056	0.00056	50.063	0.0001
Error	33	0.00037	1*10e-5		
Total	34			$R^2 = 0.6027$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.873642	0.0014617	597.689	0.0001
t-1	33	3.97*10e-4	5.68*10e-5	7.076	0.0001

Figure 5.10(b) Regressing $Tr_{BBB}(t-1)$ on t-1

Dependent Variable: $Tr_{BBB}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.02016	0.02016	58.72	0.0001
Error	33	0.01133	0.00034		
Total	34	0.031		$R^2 = 0.6402$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.688982	0.00807473	85.326	0.0001
t-1	33	0.002376	3.1*10e-4	7.663	0.0001

Figure 5.10 Results from regressing (a) $Tr_{GGG}(t-1)$ and $Tr_{BBB}(t-1)$ on t-1 - [≥5]-Mover

Figure 5.10(a) and (b) show the ANOVA and regression outputs of regressing $Tr_{GGG}(t-1)$ and $Tr_{BBB}(t-1)$ on t-1 alone respectively. The F-statistic in both cases confirmed that at 1% significance level there is a significant overall regression effect between the dependent variables and t-1.

Figure 5.11(a) Regressing $Tr_{GGG}(t-1)$ on av_r

Dependent Variable: $Tr_{GGG}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00024	0.00024	11.337	0.0019
Error	33	0.0007	2*10e-5		
Total	34	0.00093		$R^2 = 0.2557$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.906389	0.00693852	130.632	0.0001
av_r	33	-0.003566	0.00105902	-3.367	0.0019

Figure 5.11(b) Regressing $Tr_{BBB}(t-1)$ on av_r

Dependent Variable: $Tr_{BBB}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	1	0.00754	0.00754	10.392	0.0028
Error	33	0.02395	0.00073		
Total	34	0.03149		$R^2 = 0.2395$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	33	0.876436	0.04071379	21.527	0.0001
av_r	33	-0.020032	0.00621412	-3.224	0.0028

Figure 5.11 Results from regressing (a) $Tr_{GGG}(t-1)$ and (b) $Tr_{BBB}(t-1)$ on av_r - [≥5]-Mover

Figure 5.11(a) and (b) show the ANOVA and regression outputs of regressing $Tr_{GGG}(t-1)$ and $Tr_{BBB}(t-1)$ on av_r alone respectively. The F-statistic in both cases rejected that at 1% significance level there is a no overall regression effect between the dependent variables and av_r.

Figure 5.12(a) Regressing $Tr_{GGG}(t-1)$ on t-1 AND av_r

Dependent Variable: $Tr_{GGG}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	2	0.00087	0.00043	214.023	0.0001
Error	32	6.5*10e-5	2*10e-6		
Total	34	0.00093		$R^2 = 0.9304$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	32	0.899427	0.00218995	410.707	0.0001
t-1	32	0.000422	2.4*10e-5	17.619	0.0001
av_r	32	-0.004051	3.3*10e-4	-12.279	0.0001

Figure 5.12(b) Regressing $Tr_{BBB}(t-1)$ on t-1 AND av_r

Dependent Variable: $Tr_{BBB}(t-1)$		Analysis of Variance			
Source	DF	Sum of Squares	Mean Squares	F Value	Prob > F
regression	2	0.02997	0.01498	315.323	0.0001
Error	32	0.00152	5*10e-4		
Total	34	0.03149		$R^2 = 0.9517$	
		Parameter Estimates			
Variable	DF	Parameter Estimate (b_i)	Standard Error	T for H_o:b_i=0	Prob > |T|
Intercept	32	0.834915	0.01059232	78.823	0.0001
t-1	32	0.002515	1.2*10e-4	21.724	0.0001
av_r	32	-0.022926	0.00159573	-14.367	0.0001

Figure 5.12 Multiple regression results from regressing (a) $Tr_{GGG}(t-1)$ and (b) $Tr_{BBB}(t-1)$ on t-1 AND av_r collectively - [≥5]-Mover

For both $Tr_{GGG}(t-1)$ (Figure 5.12(a)) and $Tr_{BBB}(t-1)$ (Figure 5.12(b)) , the results showed that $Tr_{hij}(t-1) = a_{hij} + b_{hij}*(t-1) + c_{hij}av\_r$ is a significant regression model at 1% significance level. Each of the explanatory variables is significantly different from zero at 1% significance level (two-sided test) in the presence of all other variables in the model. In both cases, over 90% (the correlation coefficient, R^2) of the variance in the dependent variable can be explained by the linear model with t-1 and av_r.

Figure 5.13(a)

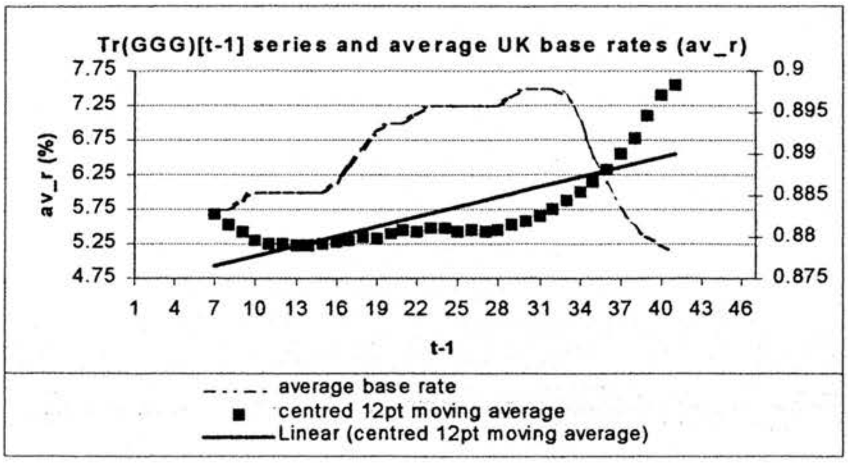


Figure 5.13(b)

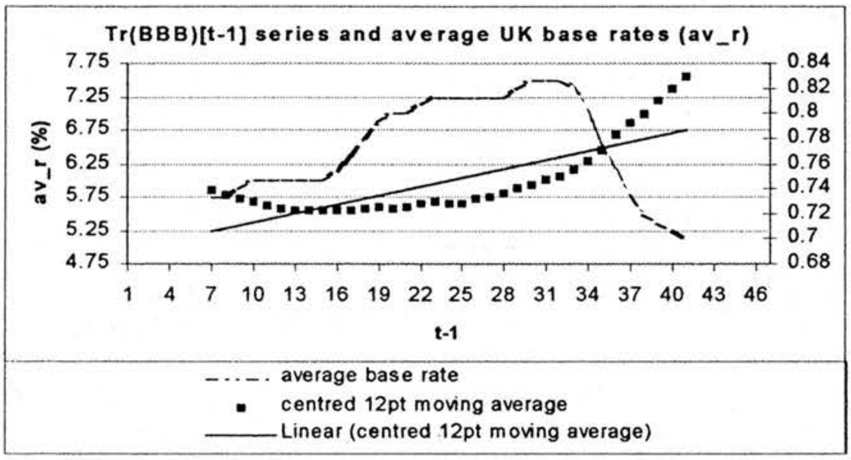


Figure 5.13 (a) Tr_{GGG}(t-1) and (b) Tr_{BBB}(t-1) series superimposed with average UK base rates av_r - [≥5]-Mover

Figure 5.13(a) shows the Tr_{GGG}(t-1) series, the $Tr_{GGG}(t-1) = a_{hij} + b_{hij}*(t-1)$ line and average base rates (av_r) on the identical time axis. The smoothed series dipped below the $Tr_{GGG}(t-1) = a_{hij} + b_{hij}*(t-1)$ line when the average rates were high and surged above it when the average rates were low. The same happened to the Tr_{BBB}(t-1) series in Figure 5.13(b). These results suggest that in general customers here (remember they were all Movers and no Stayers were involved in this chapter) who took credit found it difficult to maintain Good consecutively when interest rates were high. On the other hand they were not prepared to suffer unnecessary high interest payments.

Overall it can be concluded that the methodology of time series decomposition and linear regression applied equally well during the model fitting phase given the original and subset samples, and the original and new state-space.

5.6 Fitting the segments

From sections 5.1 to 5.5, arguments and justifications were put forward as to the reasons it was necessary to look for a new state-space and a segmentation scheme that would improve on the Markov hypothesis. Consistent methodology was necessary for comparison, as to how well the new state-space and segmentation performed relative to the old original ones. A regression forecast model was deduced which fitted $Tr_{hij}(t\text{-}1,av\_r)$ equally well with time and interest rates as explanatory variables. In this section the mechanism from section 4.6 is modified and applied to each of the optimal subsets identified a little earlier. There were more than one status that customers could move to from a given status even with a reduced state space.

Earlier in section 5.3, the order of the Markov assumption and the segmentation that is most appropriate for modelling the dynamics of the given consumer credit data was examined. It was concluded that it was most appropriate to segment the Movers(2) sample into 3 segments: [1-to-3]-, [4]- and [≥5]-Movers, and each should be modelled as a Second Order Markov Chain. So the model presented in section 4.1 is modified as follows:

Let account performance/delinquency status be i = G, I, B.

Let the month end at which the value of i was observed be $t = 1, 2, \ldots, T$.

Let $N_{hij}(t-1)$ be the transition frequency of an account being in status j at time t+1, given being in status i at time t and status h at time t-1, for i, j = G, I, B; t-1 = 1, 2, ..., (T-2).

Let $P_{hij}(t-1)$ be the transition probability of an account being in status j at time t+1, given being in status i at time t and status h at time t-1, for i, j = G, I, B; t-1 = 1, 2, ..., (T-2).

Let av_r be the average values of the UK base rates between time t-1 and t.

So, $P_{hij}(t-1, av\_r)$ is the probability of an account being in status j at t+1 given it was in status h at time t-1 and status i at time t, where the average value of base rates during the period between t-1 and t was av_r.

Equation 4.2 of section 4.6 was applied to each of the segments identified with the above quantities substituted in accordingly, to estimate the parameters that best fit the forecast model while satisfying all the constraints. Recalling from Equation 3.5 in section 3.8.3, one estimated the trend-cycle (Tr_{hij}) component of a P_{hij} series here.

5.6.1 Estimated parameters for each segment and economic impacts on Tr_{hij}

Figure 5.14, 5.15 and 5.16 display the optimal parameters estimated for [1-to-3]-, [4], and [≥5]-Mover subset, respectively, by solving Equation 4.2 given the new quantities for fitting a Second Order Markov Chain. As before the $a_{hij}(t-1, av\_r)$ parameters were the base value of a $Tr_{hij}(t-1, av\_r)$ series; the $b_{hij}(t-1, av\_r)$ and $c_{hij}(t-1, av\_r)$ parameter expressed the influence of time and interest rates on a $Tr_{hij}(t-1, av\_r)$ series respectively. The "total" columns on the right hand side in these figures confirm the constraints imposed were satisfied.

Figure 5.14(a) optimal a_{hij}(t-1,av_r)-parameter matrix - [1-to-3]-Mover subset

	j			
hi	G	I	B	total
GG	0.95617	0.04331	0.000522	1
IG	0.90858	0.09069	0.00073	1
BG	0.96313	0.01739	0.019474	1
GI	0.20674	0.764806	0.028453	1
II	0.20506	0.791087	0.003849	1
BI	0.14887	0.026104	0.825023	1
GB	0.15471	0.25656	0.58873	1
IB	0.02468	0.03694	0.93839	1
BB	0.07163	0.03242	0.89595	1

Figure5.14(b) optimal b_{hij}(t-1,av_r)-parameter matrix - [1-to-3]-Mover subset

	j			
hi	G	I	B	total
GG	8.79E-06	-1E-05	5.62E-06	0
IG	0.00027	-0.0003	8.27E-06	-1E-09
BG	0.00043	-0.0004	-5.8E-05	0
GI	0.00086	-0.00114	0.000281	1E-09
II	-0.00217	0.002182	-8.76E-06	3E-09
BI	-0.00216	0.011483	-0.00932	0
GB	-0.00329	-0.00299	0.00628	0
IB	-0.00053	-0.00019	0.00072	1E-09
BB	-0.00071	-0.00042	0.00113	2E-09

Figure 5.14(c) optimal c_{hij}(t-1,av_r)-parameter matrix - [1-to-3]-Mover subset

	j			
hi	G	I	B	total
GG	0.003	-0.0029	-7E-05	2.4E-19
IG	0.00984	-0.0097	-9.7E-05	5.1E-19
BG	-0.0005	0.00258	-0.00204	0
GI	0.02861	-0.02498	-0.00362	0
II	-0.01345	0.013834	-0.00039	0
BI	-0.00587	0.057456	-0.05159	0
GB	0.02147	-0.0155	-0.00598	0
IB	0.04402	-0.00175	-0.04228	0
BB	-0.00193	-0.00167	0.0036	0

Figure 5.14 Optimal parameters – [1-to-3]-Mover subset

Figure 5.15(a) optimal a_{hij}(t-1,av_r)-parameter matrix - [4]-Mover subset

	j			
hi	G	I	B	total
GG	0.94332	0.05579	0.000891	1
IG	0.83234	0.16696	0.0007	1
BG	0.71663	0.21814	0.065231	1
GI	0.391103	0.569701	0.039196	1
II	0.274678	0.71128	0.014043	1
BI	0.299911	0.310102	0.389987	1
GB	0.302603	-0.000326	0.697723	1
IB	0.144908	0.097188	0.757904	1
BB	0.080922	0.065753	0.853325	1

Figure 5.15(b) optimal b_{hij}(t-1,av_r)-parameter matrix - [4]-Mover subset

	j			
hi	G	I	B	total
GG	0.0005	-0.0005	7.38E-06	2.63E-20
IG	0.00045	-0.0005	1.67E-05	-3E-10
BG	0.00111	-0.0013	0.000196	0
GI	0.002806	-0.00315	0.00034	0
II	-0.001179	0.001135	4.41E-05	2E-10
BI	0.000595	-0.00045	-0.00015	0
GB	-0.0038	0.000326	0.003475	0
IB	-0.00308	-0.002068	0.005151	1E-09
BB	-0.00139	-0.001266	0.002661	0

Figure 5.15(c) optimal c_{hij}(t-1,av_r)-parameter matrix - [4]-Mover subset

	j			
hi	G	I	B	total
GG	-0.0002	0.0003	-0.0001	0
IG	0.01068	-0.0106	-4.3E-05	-1E-09
BG	0.025	-0.0163	-0.0087	0
GI	0.007983	-0.00276	-0.00523	0
II	-0.008602	0.010196	-0.00159	0
BI	-0.019958	0.069732	-0.04977	0
GB	0.033346	0.008895	-0.04224	0
IB	0.010569	0.025611	-0.03618	0
BB	0.003836	0.001032	-0.00487	0

Figure 5.15 Optimal parameters – [4]-Mover subset

Figure 5.16(a) optimal a_{hij}(t-1,av_r)-parameter matrix - [≥5]-Mover subset

	j			
hi	G	I	B	total
GG	0.92152	0.07703	0.00145	1
IG	0.72525	0.27003	0.00473	1
BG	0.72357	0.26106	0.01537	1
GI	0.40519	0.54939	0.045418	1
II	0.21133	0.78359	0.005077	1
BI	0.35192	0.61667	0.031418	1
GB	0.34566	0.11107	0.54327	1
IB	0.23236	0.0952	0.67244	1
BB	0.05897	0.07905	0.86198	1

Figure 5.16(b) optimal b_{hij}(t-1,av_r)-parameter matrix - [≥5]-Mover subset

	j			
hi	G	I	B	total
GG	0.00035	-0.0004	3.78E-05	-1E-10
IG	0.00049	-0.0005	3.19E-05	3E-10
BG	0.00073	-0.0011	0.00042	0
GI	0.00038	-0.00073	0.000356	0
II	-0.0012	0.0012	3.55E-05	1E-10
BI	0.00032	-0.00078	0.000462	0
GB	-0.00408	-0.00236	0.00644	-1E-09
IB	-0.00111	-0.00068	0.00179	-1E-09
BB	-0.00125	-0.00168	0.00294	0

Figure 5.16(c) optimal c_{hij}(t-1,av_r)-parameter matrix - [≥5]-Mover subset

	j			
hi	G	I	B	total
GG	-0.007	0.00703	-6E-05	-6.7E-19
IG	0.00265	-0.0024	-0.0003	0
BG	-0.0047	0.004	0.00067	1E-09
GI	0.00555	-0.00359	-0.00197	0
II	0.01159	-0.0144	0.002818	0
BI	-0.0213	-0.00042	0.021715	0
GB	0.01319	0.01734	-0.03053	0
IB	0.00735	0.0157	-0.02305	1E-09
BB	0.01743	0.00974	-0.02717	1E-09

Figure 5.16 Optimal parameters – [≥5]-Mover subset

Noticeably interest rates had the opposite impact on the less frequent Mover subset, i.e. [1-to-3]-Mover, than on the more frequent Mover subset, i.e. [≥5]-Mover. Take Tr_{GGG}(t-1,av_r), Tr_{BBG}(t-1,av_r) and Tr_{BBB}(t-1,av_r) for example. Interest rates had a positive effect on Tr_{GGG}(t-1,av_r) and Tr_{BBB}(t-1,av_r) of [1-to-3]-Mover subset but a negative one on [≥5]-Mover subset given the same series. The opposite happened however when one considers Tr_{BBG}(t-1,av_r), Figure 5.14(c), 5.15(c) and 5.16(c). This suggests those accounts who made frequent transitions were more sensitive to interest rate changes as they were not prepared to suffer high interest payments while making full use of the facilities given. On the other hand to those accounts which made the occasional cross-status transitions, there seemed two different groups of account behaviour. One group were those who reacted sensibly to rises in interest rates and remained Good; the other were those who had already fallen behind and found themselves trapped in delinquency with rising interest rates.

To summarise, it can be concluded in the Movers(2) sample given the new state-space and the new segmentation, there seemed to exist three distinctive groups of account behaviour: the Sensible, the Adventurer and the Incompetent.

5.6.2 Estimated P_{hij}(t-1,av_r)

In this section the accuracy and robustness of the forecast model will be examined. As illustration only P_{GGG}(t-1,av_r) and P_{BBB}(t-1,av_r) of the [≥5]-Mover subset will be used. As with its parent Movers(2) sample, the holdout method used on the optimal subset was a "post sample" method. The test set was collected after the development data set, both had identical and matched cases of accounts.

t-1	Estimated P_{GGG}(t-1,av_r)	Actual P_{GGG}	% error	Estimated P_{BBB}(t-1,av_r)	Actual P_{BBB}	% error	Estimated Tr_{GGG}(t-1,av_r)	Estimated Tr_{BBB}(t-1,av_r)
49	0.89345	0.93351	-4.29	0.84368	0.90018	-6.28	0.89756	0.84627
50	0.90066	0.93588	-3.76	0.80876	0.9235	-10.38	0.89703	0.84581
51	0.90417	0.92857	-2.63	0.84534	0.93151	-9.25	0.89738	0.84875
52	0.89329	0.93352	-4.31	0.86084	0.93995	-8.42	0.89773	0.85268
53	0.90332	0.93976	-3.88	0.82923	0.93498	-11.31	0.89807	0.85462
54	0.89785	0.93412	-3.88	0.84151	0.94459	-10.91	0.89842	0.85756
55	0.89111	sys. error	------	0.88704	sys. error	------	0.89877	0.86049
56	0.90359	sys. error	------	0.86446	sys. error	------	0.89911	0.86343
57	0.90734	sys. error	------	0.86643	sys. error	------	0.89946	0.86637
58	0.88320	0.93807	-5.85	0.89347	0.95018	-5.97	0.89981	0.86930
59	0.90273	0.93808	-3.77	0.87780	0.95644	-8.22	0.90050	0.87220

Figure 5.17 Estimated and actual P_{GGG} and P_{BBB} for [≥5]-Mover subset (seasonally adjusted, "sys. error" – system error)

Figure 5.17 displays the actual and estimated P_{GGG} and P_{BBB} series of the [≥5]-Mover subset. As reported in section 4.7.2 the field that records the performance data of the month September 2000 was not populated. As a result any status h, i, and j that involves this particular month were not observed (hence system error). Figure 5.17 shows the percentage errors are of acceptable range though it fared a little worse at the bottom end of the delinquency spectrum. In this case, the holdout sample outperformed the estimation given the P_{GGG} and P_{BBB} series for the [≥5]-Mover subset in the test period. And the large forecast errors of some were due to the strong seasonal adjustments (Sn(t-1)). It looked as if seasonal adjustment also had a trend. It was large in some of the months in the first two years of the study period (1996 and 1997), which got smaller over time. Models allowing for this could be very useful but this is out of the scope of the thesis. Therefore the estimated Tr_{GGG}(t-1,av_r) and Tr_{BBB}(t-1,av_r) of the [≥5]-Mover subset were displayed in the last two columns in Figure 5.17 for comparison.

5.7 Interpretation of results – the Sensible, the Adventurer and the Incompetent

Full range of estimated Tr_{hij}(t-1,av_r) for each of the Mover subsets at selected t-1 were listed in Appendix 9.2.3, 9.2.4 and 9.2.5. A full list of the actual P_{hij} at identical t-1 for each of the Mover subsets was listed in Appendix 9.3.3, 9.3.4 and 9.3.5. The particular values of t-1 were carefully selected to fairly reflect the estimated Tr_{hij}(t-1,av_r) at different times of a year at different years. One did experience anomalies where the estimated Tr_{hij}(t-1,av_r) fell outside the expected range of 0 and 1. The same arguments put forward in section 4.7.3 can be used here to explain such anomalies. Namely that the magnitude of T was much larger than R. As before (section 4.7.3) one can interpret the estimated Tr_{hij}(t-1,av_r) results in terms of the impact time and average interest rates had. But this was already discussed a little earlier in section 5.6.1. There is however a third way to interpret the results. This is by comparing the magnitude of estimated transition probabilities between subsets for a given Tr_{hij}(t-1,av_r) series. This is not introduced in section 4.7.3 because it would not be a fair comparison since the samples in question were different. Namely account closures were allowed in Movers(1) but not in Movers(2) This is not the case here. The subsets identified in this chapter all originated from an identical parent sample, i.e. Movers(2), and they differ from each other only in the number of transitions made. Therefore such comparison offers a genuine assessment on different behavioural classes.

The estimated Tr_{GGG}(t-1,av_r) of [1-to-3]-Mover were significantly higher than that of [≥5]-Mover, this is understandable. But the same applies to Tr_{BBB}(t-1,av_r). Furthermore the Tr_{BBG}(t-1,av_r) of [1-to-3]-Mover were roughly only one third of that of [≥5]-Mover. This suggests the less frequent Movers once they reached Bad status they are more likely to dwell on being seriously delinquent, even if they managed to crawl out of bad debt they are unlikely to fully recover in one single transition, than their more frequent counterparts.

These findings coincide with the identification of the Sensible, the Adventurer and the Incompetent earlier in section 5.6.1. The Sensible are those who are sensitive to interest rates changes and remain good in the long term. The Adventurers on the other hand would make full use of the facilities given but sensible enough not to suffer high interest payments on bad debt in rising interest rates. The Incompetent are those who once fell delinquent got trapped further in particular when interest rates are rising. To summarise, there existed three different types of behaviour in the Movers(2) sample, they were identified by segmenting the sample on the frequency of transitions given the new state-space. The Sensible and the Incompetent are low level movers, the Adventurers are high level movers. The terms the Sensible, the Adventurer and the Incompetent do not reflect or bear any relation to any official descriptions of customers.

Finally, provided a customer is a frequent Mover his chances of remaining Good at time t+1 given he was Good at time t and t-1 decrease with the number of transitions made. On the other hand, he is less likely than his less frequent counterparts to get trapped in Bad at t+1 given he was Bad at t and t-1. In the long term this presents an interesting scenario. One could jump from Bad to Good in one move, on the other hand one most certainly deteriorates to Bad through Indeterminate. In the long run provided a customer is not behaving sensibly but not bad enough to be written off, then the middle ground of the delinquency spectrum is the place he/she will revolve around. That is, a sample cohort like the ones used here would approach population homogeneity eventually.

5.8 Conclusion

The aim of this chapter was to create a new state-space or new subsets or both in order to make the Movers(2) sample conform to Markovity. From the outset it was assumed that complete Markovity was not possible. Though possible in theory, proving so would bring little benefits in a practical sense. Thus a "best approximate" was sought similar to the methodology according to Weiss et al (1982). This was the other aim of this chapter: turning a lengthy and tedious task into a manageable one and yet remain theoretically sound. In order to see how successful this procedure was in converging the Movers(2) sample to Markovity given the new state-space and/or new subsets it was necessary to keep the methodology consistent during the modelling phase.

The new, reduced state-space was the first step towards achieving the latter aim, because one could not deal practically with higher order Markov Chains unless the basic state space was smaller. The choice of states in the reduced state space was governed by the bank's experience. They had identified three super states: Good(G), Indeterminate(I) and Bad(B), which reflected the concern they had for accounts in the different states. This seemed an obvious reduction to start with. Previous knowledge on the Movers(2) sample led to sub-dividing it into sub-populations based on the number of transitions made during the study period given the new-state-space. Repetitive testing proved future transition was conditionally dependent on current given the immediate past performance status, i.e. a Second order Markov Chain. Via a new statistic, an optimal split on the Movers(2) sample produced four subsets which made up the parent sample – [0]-, [1-to-3]-, [4]-, and [≥5]- Movers subsets. Modelling on the [≥5]-Mover subset revealed a combination of time and average interest rates was adequate to explain the changes in its P_{hij} series.

Schniederjans and Loch (1994) had complained that "lumping" states together generalised the results Markov analysis provided. Similar findings can be seen in Figure 5.8 and 5.9. The new state-space was a product of amalgamating similar fine PROBE states together to form a broad performance category. The seasonal fluctuations in the figures were smaller in magnitude, though one could argue this was the result of more past history being considered. Another experience this thesis shared with the Schniederjans and Loch (1994) study was the occurrence of zero and sparse entries to transition matrices. Even in a reduced state-space, a large sample size as in here cannot escape such phenomenon, since the state-space grew exponentially when more past history was incorporated.

Nevertheless given the new state-space and subsets, a satisfactory forecasting model was derived in this chapter. The forecasting model fitted to each of the subsets performed well.

In interpreting and comparing the results of the estimated transition probabilities of the subsets, three distinctive types of customers were identified in the Movers(2) sample: the Sensible, the Adventurer and the Incompetent; each represents a distinctive type of behaviour. And it was deduced a sample cohort would reach homogeneity in a long run.

6. Multiple split measurement analysis

In Chapter 5 it was shown that segmentation improved the forecasting of the customers' delinquency dynamics. However to use these models lenders must identify to which segment based on the dynamic behaviour a customer should belong. So it would be immensely useful and informative to lenders to be able to correctly identify the different types of account behaviour during the application or performance assessment stage. This returns to Credit and Behavioural Scoring once again. In this chapter, pilot scorecards will be constructed to see whether application, or behaviour details of a customer, or both could reveal some tell-tale signs of the likely behaviour that would follow in the future. They were experimental because the classification criteria here deviated from the normal Good/Bad split criterion that is commonly used in the industry today. Furthermore data from credit reference bureau was not available in this case, which most lenders would gather. As a result the resultant scorecards might appear rudimentary. As long they met the basic theoretical requirements, no sophistication was sought. It was simply out of the scope of this thesis though one was interested in how accurately they can predict.

In the next few sections the techniques used in scorecard construction and the means of evaluating the classification rules they produced will be briefly introduced. The method of generating the training sample on which the classification rules were fitted, and the test sample on which the classification rules were tested will be discussed. Finally the results will be presented and interpreted. The methods and techniques chosen here were those most commonly employed in the industry, there might exist better variations but this is entirely another area of empirical research and out of the scope of this thesis. The scorecards developed here do not reflect or bear any relation to the scorecards or methodologies used by the RLS.

6.1 Classification criteria

As just mentioned, the classification criteria here was not the norm used in the industry. Customers are classified according to the dynamics of their delinquency status rather than their likelihood of default. And this behavioural class was defined by the number of transitions made in the study period given the new state-space (Chapter 5).

There were four of these behavioural classes. In general, from Chapter 5 it was accepted that there was a spectrum of "Movers" in the 4-Year-Full-History sample (Figure 3.1, section 3.2) based on the number of transitions made given the new state-space, ranging from low to high transition frequency. They were [0]-Mover, [1-to-3]-Mover, [4]-Mover and [≥5]-Mover, as identified in section 5.1.2. So far the [0]-Mover class was omitted from analysis in Chapter 5 as they represented those customers who made all their transitions within one performance status. But by definition presented in section 3.7, they were definitely a "Mover". As a result it was necessary to include this class in building a scorecard. One could argue these four classes were all Movers, how could one identify Stayers? The answer lies in the definition of the new state-space. Stayers were those who stayed current (PROBE state = 1) throughout the study period given the original fine detailed state-space (section 3.6). However given the new broad state-space, Stayers were also those who stayed Good (PROBE status = G) throughout the study period, hence [0]-Mover. Therefore the Stayer(2) sample was combined with the [0]-Mover sample for this scorecard building exercise. Figure 6.1 illustrates the four behavioural classes to be investigated.

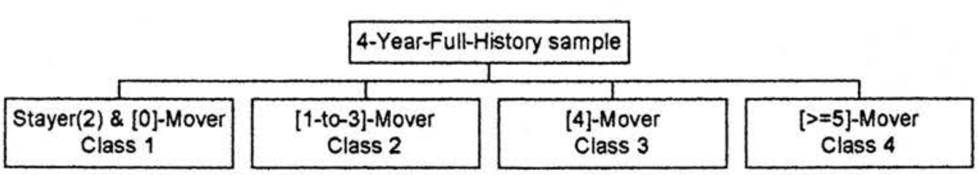


Figure 6.1 The four behavioural classes to be identified

6.2 Classification technique to build scorecards

The classification technique chosen here is commonly employed in scorecard development in the industry today. It is logistic regression. It is conceptually simple and widely available in many statistical software packages. In this context, it has the form:

$$\log[p_v/(1-p_v)] = c_v + q_1X_1 + q_2X_2 + \ldots + q_mX_m \qquad \textbf{Equation 6.1}$$

where p_v (not to be confused with the capital P which denotes transition probability throughout in this thesis) is the probability of customer X belonging to behavioural class v, who takes various values in different application or behavioural attributes X_1, X_2, …, X_m.

This is equivalent to a linear score function where the coefficients and the numeric value of the attribute are combined together and summed up in a single score (the left hand side of Equation 6.1, which is also called the "log odds" of belonging to class v). This score is a measure of relative dynamics associated to a particular behavioural class in this case. And each behavioural class should have a specific range of scores associated.

In this case there were four classes along a spectrum of increasing dynamics of delinquency, and a customer in the 4-Year-Full History must belong to one of these four classes, Figure 6.2, where $c_1 < c_2 < c_3$.

Dynamics	0 moves	1, 2, 3 moves	4 moves	≥5 moves	
Class v	1	2	3	4	
$p_v = 0$	c_1	c_2	c_3		$p_v = 1$

Figure 6.2 Behavioural classes along the delinquency dynamics spectrum

From Equation 6.1:

$p_v(v|X) = \exp(c_v + q_1X_1 + q_2X_2 + \ldots + q_mX_m)/[1+ \exp(c_v + q_1X_1 + q_2X_2 + \ldots + q_mX_m]$

and p_v must have a range between 0 and 1; this gives:

$\mathbf{p_v(v=1|X) = \exp(c_1 + q_1X_1 + q_2X_2 + \ldots + q_mX_m)/[1+ \exp(c_1 + q_1X_1 + q_2X_2 + \ldots + q_mX_m]}$ **Equation 6.2**

$p_v(v\leq 2|X) = \exp(c_2 + q_1X_1 + q_2X_2 + \ldots + q_mX_m)/[1+ \exp(c_2 + q_1X_1 + q_2X_2 + \ldots + q_mX_m]$

$\Rightarrow$

$\mathbf{p_v(v=2|X) = p_v(v\leq 2|X) - p_v(v=1|X)}$ **Equation 6.3**

$= \exp(c_2 + q_1X_1 + q_2X_2 + \ldots + q_mX_m)/[1+ \exp(c_2 + q_1X_1 + q_2X_2 + \ldots + q_mX_m] - \exp(c_1 + q_1X_1 + q_2X_2 + \ldots + q_mX_m)/[1+ \exp(c_1 + q_1X_1 + q_2X_2 + \ldots + q_mX_m]$

Since $c_2 > c_1 \Rightarrow \exp(c_2) > \exp(c_1)$

Hence $[\exp(c_2)\exp(\mathbf{qX}) + \exp(c_2)\exp(c_1)\exp(2\mathbf{qX})] >$
$[\exp(c_1)\exp(\mathbf{qX}) + \exp(c_1)\exp(c_2)\exp(2\mathbf{qX})]$

Therefore

$\{\exp(c_2)\exp(\mathbf{qX}) / [1+\exp(c_2)\exp(\mathbf{qX})]\} > \{\exp(c_1)\exp(\mathbf{qX}) / [1+\exp(c_1)\exp(\mathbf{qX})]\}$

i.e. $p_v(v\leq 2|X) > p_v(v=1|X)$

$\Rightarrow \mathbf{p_v(v=3|X) = p_v(v\leq 3|X) - p_v(v\leq 2|X)}$ **Equation 6.4**

$\Rightarrow p_v(v=4|X) = 1 - p_v(v \leq 3|X)$ **Equation 6.5**

So Equation 6.2, 6.3, 6.4 and 6.5 give the respective probabilities of a customer in the 4-Year-Full-History sample belonging to one of the four behavioural classes given his/her application or behavioural attributes.

Then along this spectrum of dynamics there were six pairs of comparisons on the scores attained in each class given the four classes in Figure 6.2. These were Class 1 against Class 2, 3 and 4; Class 2 against 3 and 4; then finally Class 3 against 4.

6.3 Assessment of classification rules

The chosen assessment method here was also the most common method used for evaluating scorecards in the industry today. It is a "Receiver Operating Characteristic (ROC)" curve, which is sometimes referred as a Lorenz diagram. In terms of Credit Scoring, each point on the ROC curve corresponds to rejecting a specific proportion of the population. The horizontal value is the proportion of the good population who were rejected and the vertical value is the proportion of the bad population who were rejected. Initially when no one is rejected they both have a value of zero and eventually when everyone is rejected they are both one. A ROC curve is usually plotted at a specific credit score. Thus a ROC curve which is a straight line on the diagonal would be useless to lenders as the scorecard that it produced would be incapable of separating the good from the bad. A perfect ROC curve would hug the two axes. The further away from the diagonal is the ROC curve, the better it is separating the good from the bad. This means a good ROC curve would have a larger area under it. Thus it follows that a straight diagonal ROC curve would have an area of ½ under the curve, and a perfect one would have an area of 1 under the curve. This provides one with a quantitative measure where it is difficult to judge visually. In practice, most ROC curves lie in the region between the axes and the diagonal. Another quantitative measure commonly used in the industry is the Gini (G) coefficient, which is twice the area between the ROC curve and the diagonal line. Figure 6.3 displays a general ROC curve.

Here a ROC curve was plotted for each pair of comparisons across the whole range of scores attained on the test set. Each customer in the 4-Year-Full-History sample had his/her delinquency dynamics identified through the analysis in previous chapters (i.e. the value of v each customer took was known). The (**qX**) score was computed for each customer in the test set given his/her application/behavioural details. The scores were sorted in ascending order. Only those who actually were in the classes in question were extracted for plotting the ROC curve.

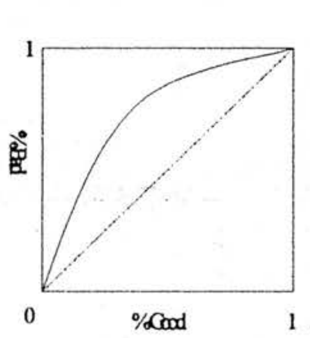


Figure 6.3 A general ROC curve in Credit Scoring

6.4 Training and test sample

6.4.1 Application scorecard

The personal attributes of the main account holding customers in the 4-Year-Full-History sample (section 3.2, Figure 3.1) whose accounts were successfully opened at the customers' own request in the month of January 1995 was collected. This means these customers went to the RBS and applied for a personal current account at their own free will. These accounts opened were not the result of transfers, servicing, renewal application, business policies, errors and so on. January 1995 was a good month for two reasons. First the existing database at the RLS was not fully operational until 1995. Second this would allow 12 calendar months for new customers to settle into their usual cash flow patterns (e.g. income, mortgage payment, direct debit, etc.) before the study period begins (i.e. January 1996).

These personal attributes were information on age, sex, marital and residential status, employment and income details and other credit commitments of the customers, which are usually requested by any lenders at the application stage.

6.4.2 Behavioural scorecard

A separate, random sample of behavioural attributes as at the month end of January 1996 on the customers in the 4-Year-Full-History sample was extracted by the RLS. January 1996 as stated before (Chapter 3) was the start of the study period. Lenders would want to determine the likely future behaviour given the past performance. In this case, this likely future behaviour was the number of transitions a customer is going to make.

These behavioural attributes include information on the behavioural score the customer attained, the overdraft limit and facilities granted, and the age of the account as at the month end of January 1996.

6.4.3 Combined scorecard

The behavioural attributes of customers from section 6.4.2 were matched to their corresponding application attributes from section 6.4.1 to form a combined set of attributes on the customers in the 4-Year-Full-History sample. This should strengthen the classifying power of the scorecard by pooling together application and behavioural attributes. This combined set of attributes contained all the personal and behavioural information described in section 6.4.1 and 6.4.2 on customers in the 4-Year-Full-History sample. The exception is that 7 characteristics corresponding to the age of the account were dropped, since every case in this sample would have the same age, i.e. 12 months.

6.5 Attribute classification, holdout method and selection procedure

Application and behavioural attributes are often categorical, like marital status, facilities granted, etc.. It is normal practice to classify both categorical and continuous attributes into binary dummy variables. This remedies the non linearity between the dependent variable and a continuous attribute. For the three samples developed, each individual value found in each categorical attribute was classified. Values of continuous attributes, like age, behavioural score, were banded together. Each band had equal width in this case, alternatively one could divide bands into equal proportions. No assumptions were made about the underlying distribution of the values taken by each attribute. One attribute value was left out for both categorical and continuous attribute to ensure non-collinearity.

The samples of classified personal and behavioural attributes were split into a training set which was used in scorecard construction, and a test set which was used to evaluate the resultant scorecard with a 70%-30% split. A stepwise procedure was imposed when performing the logistic regression analysis to ensure optimality.

However the sample of combined attributes was comparatively small to its personal and behavioural counterparts. The holdout method would not be appropriate. Instead a bootstrapped method (with placement) was employed on this particular sample. That is, the whole set was used as the training set then as the test set.

This classification and selection procedure should reveal which value (or band of values) of an attribute was most powerful in determining the future segment behaviour most appropriate for the customer. Figure 6.4 summaries the sizes and number of dummy variables developed for each case of scorecard construction.

Attributes	Size of Training set	Size of Test set	No. of dummy variables
application	1871 cases	808 cases	52
behavioural	3453 cases	1483 cases	19
combined	659 cases	659 cases	64

Figure 6.4 Summary of samples developed for scorecard construction

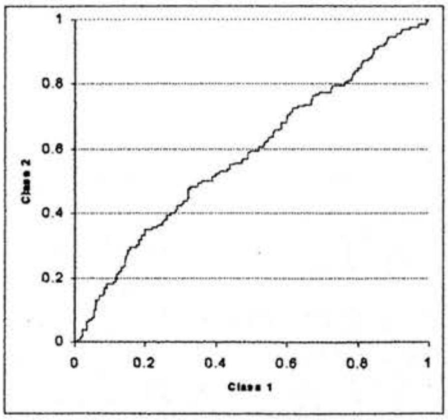
6.6 Logistic regression splitting

The PROC LOGISTIC procedure of the SAS statistical software was used to fit the logistic regression model to the data. The bootstrapped method was performed using the %BOOT macro from the same software. The PROC LOGISTIC procedure models and fits via the maximum likelihood methods a parallel lines regression model that is based on the cumulative probability distribution of the response values (i.e. Class v). By default SAS orders and models the data in ascending order of the response values.

A stepwise logistic procedure produced the following application scorecard, Figure 6.5. The left hand column provides the description of the variable selected. The middle column displays the value of coefficient associated with that variable. The right hand column gives the probability that the hypothesis of the coefficient being zero is true. Under the chosen significance level of 5%, all the coefficients were not significantly zero. That is they each contribute to the scorecard.

Description of variable	Coefficient (q_m)	p-value
c_1	-0.2112	------
c_2	0.6247	------
c_3	0.9646	------
50 year< customer age ≤ 60 year	0.7066	0.0001
customer age > 60 year	1.2559	0.0002
customer is self employed	-0.3459	0.0332
5 years < customer is in current employment ≤ 10 years	0.4601	0.0002
customer is in current employment > 10 years	0.2864	0.0207
monthly salary payment	0.4542	0.0001

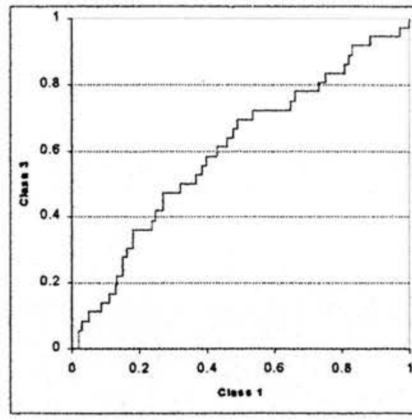
Figure 6.5 Application scorecard



Class 2 v Class 1

area under curve = 0.58009

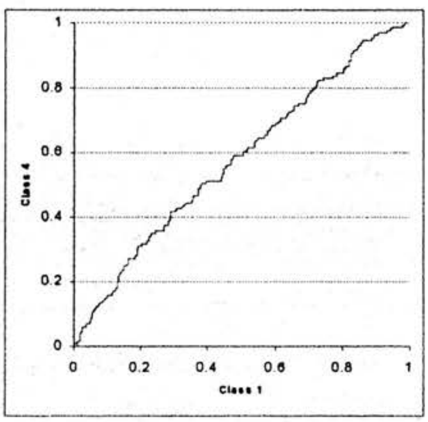
G = 0.16018



Class 3 v Class 1

area under curve = 0.60047

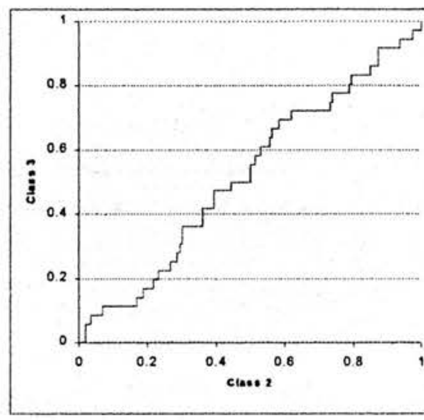
G = 0.20094



Class 4 v Class 1

area under curve = 0.57543

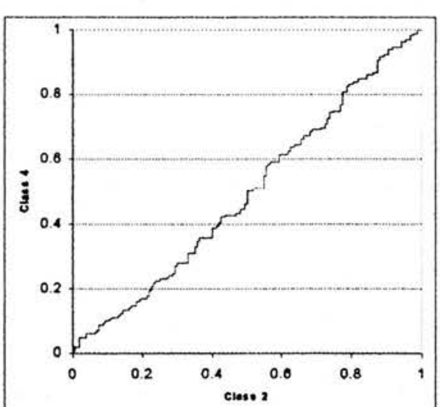
G = 0.15086



Class 3 v Class 2

area under curve = 0.51997

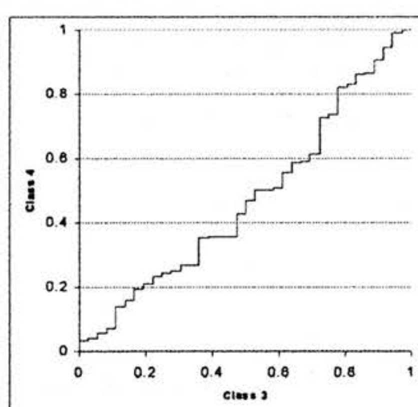
G = 0.03994



Class 4 v Class 2

area under curve = 0.49276

G = -0.01448



Class 4 v Class 3

area under curve = 0.47303

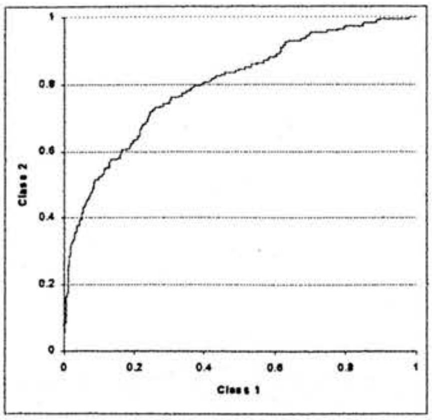
G = -0.05394

Figure 6.6 ROC curves for logistic application scorecard

A stepwise logistic procedure produced the following behavioural scorecard, Figure 6.7. Under the significance level of 5%, the hypothesis of each of the coefficients being zero was rejected.

Description of variable	Coefficient (q_m)	p-value
c_1	1.9341	------
c_2	3.0099	------
c_3	3.4001	------
behavioural score attained ≤ 600	-2.8233	0.0001
600 < behavioural score attained ≤ 700	-1.0127	0.0001
700 < behavioural score attained ≤ 800	-1.3865	0.0002
behavioural score attained > 900	0.5322	0.0063
£500 < overdraft limit ≤ £1000	0.8687	0.0001
overdraft limit > £1000	0.4650	0.0125
0 months < age of account ≤ 60 months	-0.3243	0.0062
60 months < age of account ≤ 120 months	-0.4257	0.0002
no cash card facility	-0.4643	0.0001
held a Cashline cash card	-0.7684	0.0001

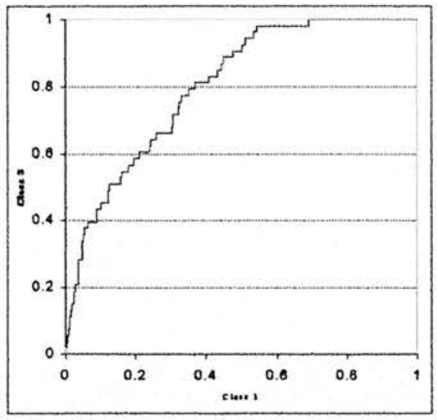
Figure 6.7 Behavioural scorecard



Class 2 v Class 1

area under curve = 0.79750

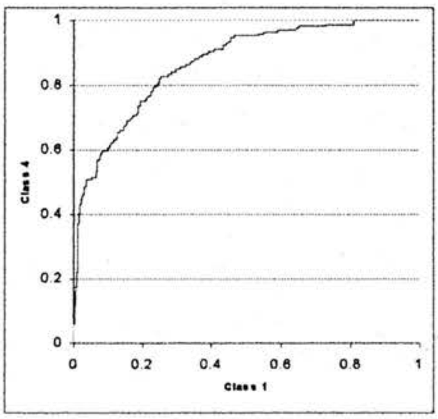
G = 0.59500



Class 3 v Class 1

area under curve = 0.80127

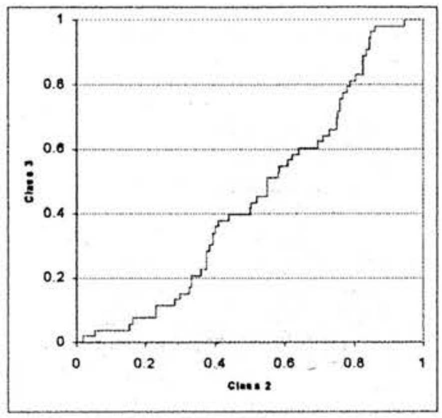
G = 0.60254



Class 4 v Class 1

area under curve = 0.86783

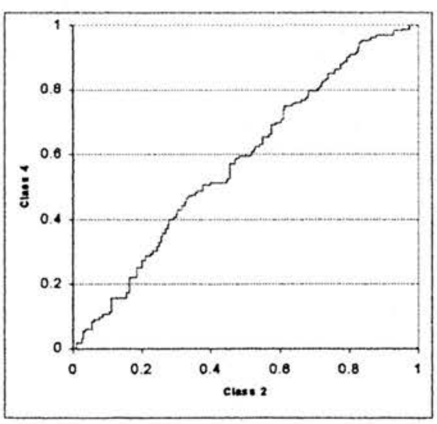
G = 0.73566



Class 3 v Class 2

area under curve = 0.44811

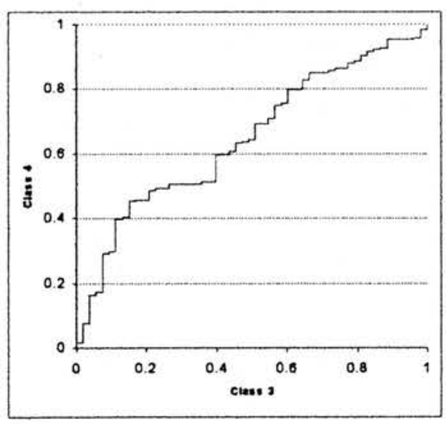
G = -0.10378



Class 4 v Class 2

area under curve = 0.57729

G = 0.15458



Class 4 v Class 3

area under curve = 0.64668

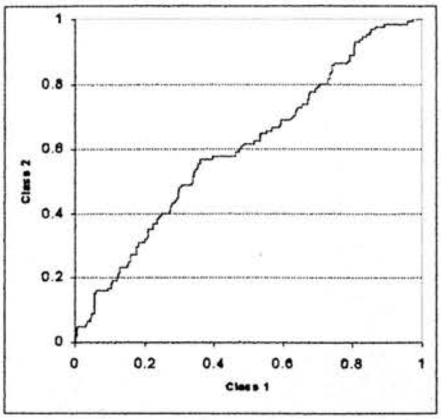
G = 0.29336

Figure 6.8 ROC curves for logistic behavioural scorecard

A bootstrapped stepwise logistic procedure produced the following combined scorecard, Figure 6.9. Under the significant level of 5%, the hypothesis of each of the coefficients being zero was rejected.

Description of variable	Coefficient (q_m)	p-value
c_1	0.7729	------
c_2	1.7649	------
c_3	2.1342	------
salary is paid in cash	-0.6337	0.0187
account located in south England	0.3906	0.0320
behavioural score attained $\leq$ 600	-1.6383	0.0001
overdraft limit = £0	-0.5358	0.0009

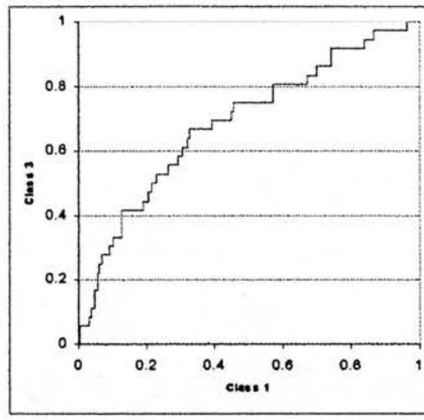
Figure 6.9 Combined scorecard



Class 2 v Class 1

area under curve = 0.60142

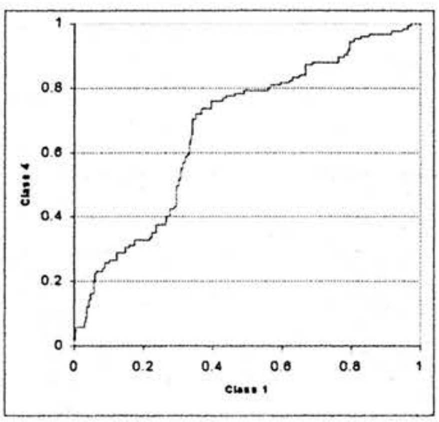
G = 0.20284



Class 3 v Class 1

area under curve = 0.68551

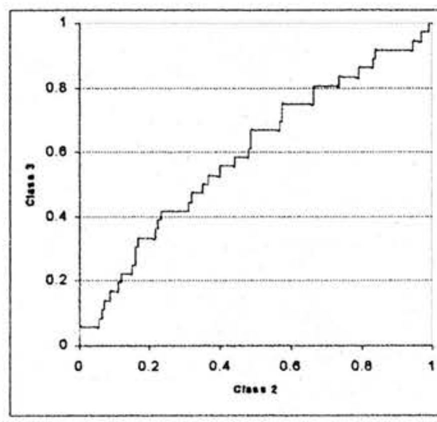
G = 0.37102



Class 4 v Class 1

area under curve = 0.67408

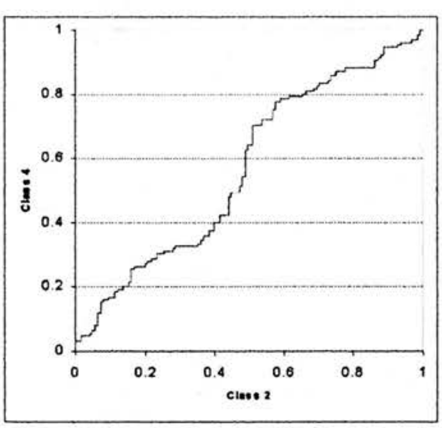
G = 0.34816



Class 3 v Class 2

area under curve = 0.59378

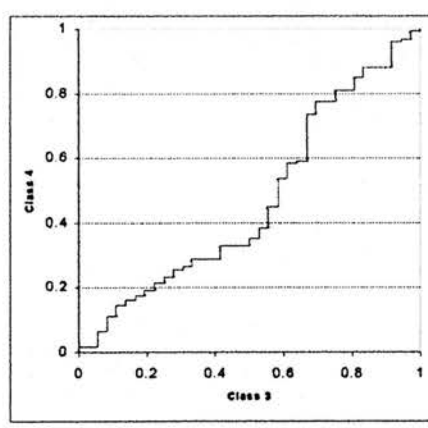
G = 0.18756



Class 4 v Class 2

area under curve = 0.56710

G = 0.13420



Class 4 v Class 3

area under curve = 0.46956

G = -0.06088

Figure 6.10 ROC curves for logistic combined scorecards

6.7 Interpretation of results

This scorecard development exercise produced some results that one would expect under normal circumstances, though the classification criteria had changed from the norm.

The scorecards developed fared better at distinguishing other behavioural classes from Class 1 than from each other. This might be due to the fact that the "true Stayer" (i.e. Stayer(2)) resided in this class. The scorecards developed found it difficult to distinguish between Class 2 ([1-to-3]-Mover) and Class 3 ([4]-Mover). The difference between the two was small, which was only a single transition given the new state space. One could try again by grouping the two classes together.

Though there were less attributes for the selection procedure to choose from, behavioural attributes by far produced the most powerful scorecard out of the three considered. The combined scorecard provided a mixed picture. Although the ROC curves generated performed better than that of the application scorecard, visually it was difficult to judge as some region fared better and some worse.

The application scorecard developed, Figure 6.5, revealed that mature applicants, in particular over 50 years of age in this case, were less likely to make frequent transitions over time. The same applied to customers who were in current employment for more than 5 years. This coincides with the traditional belief in the character and capability (2 "C"s out of the three) of the applicant in relation to his/her future performance.

The behavioural scorecard developed, Figure 6.7, revealed those customers who attained the top end of the behavioural score range were less likely to make frequent transitions. In terms of the age of the account, those who held the account between 5 to 10 years were more likely to be dynamic than those who held the account for less than 5 years. This indicates the quality of the portfolio deteriorated in a long run. Those customers with no or the lower end of cash card facilities were found to be likely to be a frequent Mover too. The behavioural scores used here were originally for Behavioural Scoring purposes. They proved in this case to be a useful attribute in identifying customers' delinquency dynamics.

The combined scorecard developed, Figure 6.9, revealed a different picture. Though the attributes used to construct this particular scorecard came from the application and behavioural side of a customer, when combined they produced a different set of variables to their application and behavioural counterparts. It revealed those customers who had their salary paid in cash, or had no overdraft limit, or attained a score at the lower end of the behavioural score range were found to be more likely to be frequent Movers. The combined scorecard was not very powerful because characteristics corresponding to the age of account was removed since each case in this sample would have identical account maturity. These characteristics were stronger predictors in the behavioural scorecard, Figure 6.7. One could improve the combined scorecard by forcing the characteristics from the behavioural scorecard into the construction of the combined scorecard.

Figures 6.6, 6.8 and 6.10 display the six ROC curves and the Gini coefficient for each of the six pairs of comparisons between the four behavioural classes described in section 6.1, for the application, behavioural and combined scorecard developed, respectively.

6.8 Conclusion

A pilot scorecard development scheme here discovered some interesting results. The intriguing part did not come from the optimal characteristics selected which were much expected.

In this pilot scheme, the methodology employed was taken from the normal practice in the consumer credit risk industry. Much research has been carried out on credit and behavioural scorecard construction using new techniques like neural networks.

However, whether a different scorecard construction technique, or whether a fifth behavioural class, namely the "true Stayers" being a class on its own, could produce better results remain out of the scope of this thesis.

The best results of the scorecards developed are those that compare Class 1 (Stayer(2) and [0]-Mover) with the other classes. Overall the results are encouraging if not usable at present. It is because the scorecards here are better than chance, they are composed of meaningful variables and the signs are pointing to the right direction. The scorecards can be improved by using data from credit reference bureau, careful choice of performance states, and segmentation on other aspects of behaviour as well as delinquency dynamics.

7. Conclusion

7.1 Achievements

This thesis consolidated existing techniques and methodologies successfully into a whole, usable forecast model on bank customers' delinquency dynamics based on a Markov Chain model. This is a model of the dynamics of consumer credit behaviour as opposed to forecasting defaults in a specified future. It challenged the unquestioned assumptions taken by previous authors of constant transition probabilities over time and a First Order Markov Chain and refuted them through formal testing. The impact of trend and interest rates on transition probability was established then successfully fitted into the Markov Chain model. This is also the first time one has tried to build explicit relationships between the transition probabilities and the economic conditions despite many long suspected such dependency.

Although it adopted a Mover-Stayer notion, the model built on the initial state space was far from Markov. A Markov Chain of higher order or a more suitable state space, or both is necessary to retain Markovity. In this case both were tried to an extent limited by practicality. One runs the risk of exponentially expanding state space in which many entries would be theoretically and physically impossible, not to mention the difficulty of sparse and zero entries off the diagonal of the state space, should one attempt the former alone. The Mover-Stayer concept was extended by further segmentation for homogeneous segments in terms of delinquency dynamics. An aggregated state space, a higher order Markov Chain and homogeneous segments put to a newly formulated test produced an optimal Second Order Markov Chain model given the new state space and segmentation. The relationship between transition probabilities and trend and interest rates for each identified segment was established and fitted into the Second Order Markov Chain successfully.

Finally, for lenders to be able to use the resultant model a pilot scorecard scheme was attempted to see whether application or behavioural details could identify the likely membership of a customer to each of the dynamic segments identified. Though the scorecards produced are not directly usable the results are encouraging and pointing to the right direction.

7.2 Future work

The resultant model as it stands can be used to forecast the number of delinquents directly. This is important as lenders would like to know what provision for bad debt they should set aside. If lenders would like to work out the costs incurred and profits gained, then an extension of the state space is required. For example the cost of being in each delinquent state needs to be estimated.

In this thesis, one has to deal with the basic state space provided by the RLS system. The advantage is that the results can be easily translated and understood to the lenders. The disadvantage is there may exist a better state space for the portfolio characteristics in question (delinquency dynamics in this case). The same argument applies to segmentation criteria. The obvious criterion to study delinquency dynamics is the frequency of movements (the one adopted in this thesis), though one could explore other avenues, maturity of accounts for example. The choice of alternatives is numerous but subjective.

A linear time series decomposition model, with the help of Pegels' (1969) classification of forecasting scenarios, was chosen from a choice of three in section 3.8.2. Other possible forms have not been explored. Similarly there are alternative functional forms between the transition probability and the economic indicators. Although our results have suggested a linear relationship between the two, one should not ignore other possibilities.

There is a pool of economic indicators in the public domain. Base rates is only one of many that are available. It was chosen because it applies to all retail banks in the UK, and it determines how much interest lenders would charge. Inclusion of base rates ruled out many potential indicators here due to collinearity, for example mortgage commitment as a percentage of income. As a result, one has to be careful when choosing further indicators on top of base rates. Not only the indicator has to be temporally compatible with the data on transition probability, it should not have a major dependence on indicators already incorporated into the model. Unemployment rate is one possibility. But one has to be careful on the choice between seasonally adjusted figures and non seasonally adjusted figures. Another possible indicator is the margin above the base rate lenders charge. This may reveal a fairer picture of the relationship between transition probability and interest rates. However this is a micro factor, the macro economic influence on customers' behaviour is the subject of interest in this thesis.

The base rate itself is a time series, and a trend exists in the base rates data used in this thesis. This contributes to the exceptionally high correlation coefficients seen in Chapter 3 and 5. To remedy this one could "de-trend" the base rates series by differencing before fitting it into the model. One argument against doing this is that The Monetary Policy Committee of The Bank of England has the authority to decide changes in base rates. That is the underlying process is not an objective one.

Finally to summarise the various aspects of possible further work:

1. incorporation of business parameters into the model, turning it from a Markov Chain model into a Markov Decision Process;
2. investigate alternative state space for delinquency dynamics;
3. investigate alternative criteria of delinquency dynamics;
4. investigate alternative functional dependency of transition probability on economic indicators;
5. investigate and incorporate alternative economic indicators into the model.

However, in all cases the development of segmentation, higher order, non-stationary and economy-dependent Markov Chain models of consumer credit behaviour has proved a useful advance to developing such models. This was the objective of this thesis.

8. References

Anderson, T. W., Goodman, L. A. (1957), "Statistical Inference about Markov Chains", Annals of Mathematical Statistics, vol. 28, pp. 89-110

Bank of England, www.bankofengland.co.uk

Blumen, I., Kogan, M., McCarthy, P. J. (1962), "The Industrial Mobility of Labor as Probability Process", Cornell Studies of Industrial and Labor Relations, vol. 6, Cornell University Press, Ithaca, N.Y..

Capon, N. (1982), "Credit Scoring Systems: A Critical Analysis", Journal of Marketing, vol. 46, pp. 82-91

Corcoran, A. W. (1978), "The use of Exponentially Smoothed Transition Matrices to Improve Forecasting of Cash Flows from Accounts Receivable", Management Science, vol. 24, no. 7, pp.732-739

Cyert, R. M., Davidson, H. J., Thompson, G. L. (1962), " Estimation of the Allowance for Doubtful Accounts by Markov Chains", Management Science, vol. 8, pp. 287-303

Cyert, R. M., Thompson, G. L. (1968), "Selecting a Portfolio of Credit Risk by Markov Chains", Journal of Business, vol. 1, pp. 39-46

Cyert, R. M., Trueblood, R. M. (1957), " Statistical Sampling Techniques in the Ageing of Account Receivable in a Department Store", Management Science, vol. 3, no. 2, pp. 185-195

Frydman, H. (1984) "Maximum Likelihood Estimation in the Mover-Stayer Model", Journal of the American Statistics Association, vol. 79, pp. 632-637

Frydman, H., Kallberg, J. G., Kao, D. L. (1985), "Testing the Adequacy of Markov Chain and Mover-Stayer Model as Representations of Credit Behaviour", Operations Research, vol. 33, no. 6, pp1203-1214

Hand, D. J. (1999), "Data Mining: New Challenges for Statisticians", Proceedings of the ASC International Conference 1999, (ed) Christie, C., Francis, J., et al, Association for Survey Computing, pp. 21-29

Hand, D. J., Henley, W. E. (1997), "Statistical Classification Methods in Consumer Credit Scoring: A Review", Journal of the Royal Statistical Society, Series A, vol. 160, part 3, pp. 523-541

Kallberg, J. G., Saunders, A. (1983), "Markov Chain Approaches to the Analysis of Payment Behaviour of Retail Credit Customers", Financial Management, vol. 12, part 2, pp. 5-14

Lewis, E. M. (1992), An Introduction to Credit Scoring, Athena Press, San Rafael, CA.

Liebman, L. H. (1972), "A Markov Decision Model for Selecting Optimal Credit Control Policies", Management Science, vol. 18, no. 10, pp. 519-525

Makridakis, S., Wheelwright, S. C, Hyndman, R. J. (1998), Forecasting Methods and Applications, U.S., John Wiley & Sons, 3rd edition

Mays, E. (1998), Credit Risk Modelling – Design and Application, Glenlake Publishing, Chicago.

McNab, H., Wynn, A. (2000), Principles and Practice of Consumer Credit Risk Management, CIB Publishing.

Metha, D. (1972), "Markov Process and Credit Collection Policy", Decision Sciences, vol. 3, pp. 27-43

Mullich, J. (1997), "Data Mining: Making Data Meaningful, Computer, vol. 30, no. 9, p. 18

Pegels, C. C. (1969), "Exponential Forecasting: Some New Variations", Management Science, vol. 12, no. 5, pp. 311-315

Pindyck, R. S., Rubinfeld, D. L. (1998), Econometric Models and Economic Forecasts, U.S., McGraw-Hill, 4th edition

Rose, P. S. (1999), Commercial Bank Management, McGraw-Hill, 4th edition

Rosenberg, E. and Gleit, A. (1994), "Quantitative Methods in Credit Management: A Survey", Operations Research, vol. 42, no. 4, pp. 589-613

Schniederjans, M. J., Loch, K. D. (1994), "An Aid for Strategic Marketing in the Banking Industry: A Markov Analysis", Computers and Operations Research, vol. 21, part 3, pp. 281-287

Smith, K. A. (1999), Introduction to Neural Networks and Data Mining for Business Applications, Australia, Eruditions Publishing

Sullivan, A. C. (1987), "Economic Factors Associated with Delinquency Rates on Consumer Instalment Debt", Working Paper No. 55, Credit Research Center, Krannert Graduate School of Management, Purdue University, U.S.

Thomas, L. C. (2000), "A Survey of Credit and Behavioural Scoring: Forecasting Financial Risk of Lending to Consumers", International Journal of Forecasting, vol. 16, pp. 149-172

Thomas, L. C., Ho, J., Scherer, W. T. (to appear), "Time will tell: Behavioural Scoring and the Dynamics of Consumer Credit Assessment", IMA Journal of Management Mathematics

Van Kuelen, J. A. M., Spronk, J., Corcoran, A. W. (1981), "On the Cyert-Davidson-Thompson Doubtful Accounts Model", Management Science, vol. 27, no. 1, pp. 108-111

Weiss, E. N., Cohen, M. A., Hershey, J. C. (1982), "An Iterative Estimation and Validation Procedure of Specification of Semi-Markov Models with Application to Hospital Patient Flow", Operations Research, vol. 30, no. 6, pp. 1082-1104

Winters, P. R. (1960), "Forecasting Sales by Exponentially Weighted Moving Averages", Management Science, vol. 6, pp. 324-342

Zandi, M. (1998), "Incorporating Economic Information into Credit Risk Underwriting", in Mays, E. (ed), Credit Risk Modelling, Glenlake Publishing, Chicago, pp. 155-168.

9. Appendix

9.1 PROBE States definitions

PROBE States Codes and Other Performance Measures

PROBE states codes are used to define customer performance at a finite level and also at a coarse level to provide flexible Good/Indeterminate/Bad definitions. PROBE state of an account is taken at each month's end. As soon as an account falls into a state it is set to that state. Each account is tested for the below states and status in the order of: 12, 11, 10, 9, 8, 7, 6, 3, 4, 2, 5, 0 and 1. For confidentiality reasons the description of each state is omitted. Throughout this thesis, a 'state' is referred as an individual performance state defined below, i.e. 0,1, 2, ..., 12. A 'status' is referred as a broad category of states defined below, i.e. Good, Indeterminate, Bad.

STATE	STATUS
0	Indeterminate
1	Good
2	Good
3	Indeterminate
4	Indeterminate
5	Indeterminate
6	Indeterminate
7	Bad
8	Bad
9	Bad
10	Good
11	Bad
12	Bad

9.2 Estimated Trend-Cycle component $Tr_{ij}(t,r(t))$ (or $Tr_{hij}(t\text{-}1,av\_r)$) for Transition Probability P_{ij} (or P_{hij})

9.2.1 Movers(1)

t	r(t)	Tr_{00}	Tr_{01}	Tr_{02}	Tr_{03}	Tr_{04}	Tr_{05}	Tr_{06}	Tr_{09}	Tr_{010}	SUM_Tr_{0j}
7	5.75	0.87249	0.075901	0.0096719	0.0004398	0.0003598	0.002331	0.0003675	3.32E-05	0.038404	1
14	6	0.87136	0.077604	0.0099705	0.000415	0.0004091	0.0023386	0.0004088	2.84E-05	0.0374662	1
21	7	0.88667	0.072327	0.0081784	0.0004109	0.0005015	0.0017568	0.0003662	2.56E-05	0.0297608	1
28	7.25	0.88554	0.07403	0.008477	0.0003861	0.0005508	0.0017644	0.0004075	2.09E-05	0.028823	1
35	6.75	0.86796	0.082712	0.0108663	0.0003407	0.0005572	0.0023617	0.0005326	1.41E-05	0.034653	1

t	Tr_{10}	Tr_{11}	Tr_{12}	Tr_{13}	Tr_{14}	Tr_{15}	Tr_{16}	Tr_{19}	Tr_{110}	SUM_Tr_{1j}
7	0.01316	0.89983	0.06481	0.007639	5.465E-05	0.0051542	0.0021938	2.08E-05	0.0071355	1
14	0.01228	0.89279	0.07253	0.007654	5.237E-05	0.0047702	0.0022287	1.54E-05	0.0076693	1
21	0.01199	0.88636	0.07909	0.007388	5.549E-05	0.0038739	0.0021286	1.04E-05	0.0091078	1
28	0.01111	0.87932	0.08681	0.007403	5.322E-05	0.0034898	0.0021635	4.96E-06	0.0096416	1
35	0.00966	0.87167	0.0957	0.007697	4.556E-05	0.0036181	0.0023333	-7.9E-07	0.0092706	1

t	Tr_{20}	Tr_{21}	Tr_{22}	Tr_{23}	Tr_{24}	Tr_{25}	Tr_{26}	Tr_{27}	Tr_{28}	Tr_{29}	Tr_{210}	SUM_Tr_{2j}
7	0.00506	0.25122	0.64943	0.029314	0.0031846	0.02907	0.01852	0.002804	0.001616	5.41E-05	9.72E-03	1
14	0.00458	0.24982	0.65525	0.028993	0.0033349	0.027176	0.01818	0.002688	0.001733	3.98E-05	8.21E-03	1
21	0.00337	0.24808	0.66534	0.028144	0.0033378	0.023362	0.01758	0.002408	0.001876	2.64E-05	6.48E-03	1
28	0.00289	0.24668	0.67116	0.027823	0.0034881	0.021468	0.01723	0.002293	0.001992	1.21E-05	4.96E-03	1
35	0.00313	0.24561	0.67271	0.028029	0.003786	0.021494	0.01716	0.002342	0.002083	-3.03E-06	3.66E-03	1

t	Tr_{31}	Tr_{32}	Tr_{33}	Tr_{34}	Tr_{35}	Tr_{36}	Tr_{37}	Tr_{39}	Tr_{310}	SUM_Tr_{3j}
7	0.29317	0.2684531	0.19766	0.000268	0.05845	0.16887	0.00471	0.00021	0.00821	1
14	0.29602	0.2765295	0.19031	0.000246	0.05358	0.17048	0.00497	0.00018	0.00769	1
21	0.27068	0.3039647	0.18742	0.000139	0.05217	0.17529	0.00356	0.00016	0.00661	1
28	0.27353	0.3120411	0.18007	0.000117	0.0473	0.17689	0.00382	0.00013	0.00609	1
35	0.30457	0.3007586	0.16826	0.000181	0.03897	0.17529	0.00575	8.6E-05	0.00614	1

t	Tr_{40}	Tr_{41}	Tr_{42}	Tr_{43}	Tr_{44}	Tr_{45}	Tr_{46}	Tr_{49}	Tr_{410}	SUM_Tr_{4j}
7	0.01699	0.07198	0.15599	0.01874	0.68078	0.00308	0.01634	0.00045	0.03564	1
14	0.01759	0.07291	0.14298	0.01532	0.69947	0.00287	0.01405	0.00048	0.03433	1
21	0.01707	0.06431	0.14285	0.0119	0.71148	0.00246	0.01277	0.00056	0.03659	1
28	0.01767	0.06524	0.12984	0.00848	0.73018	0.00225	0.01048	0.00059	0.03528	1
35	0.01939	0.07569	0.10393	0.00505	0.75555	0.00226	0.00718	0.00056	0.03039	1

t	Tr_{50}	Tr_{51}	Tr_{52}	Tr_{53}	Tr_{54}	Tr_{55}	Tr_{56}	Tr_{59}	Tr_{510}	SUM_Tr_{5j}
7	0.01018	0.17726	0.1094	0.02283	2.3E-05	0.65721	0.01335	0.00024	0.00952	1
14	0.00929	0.18026	0.12091	0.02362	3.3E-05	0.64377	0.01321	0.00019	0.00872	1
21	0.00918	0.18305	0.13342	0.02576	3.8E-05	0.62645	0.01326	0.00015	0.0087	1
28	0.00829	0.18605	0.14493	0.02655	4.8E-05	0.61301	0.01312	9.6E-05	0.0079	1
35	0.00662	0.18925	0.15547	0.02601	6.2E-05	0.60345	0.01278	3.6E-05	0.00631	1

t	Tr_{61}	Tr_{62}	Tr_{63}	Tr_{64}	Tr_{65}	Tr_{66}	Tr_{67}	Tr_{68}	Tr_{69}	Tr_{610}	SUM_Tr_{6j}
7	0.1385	0.21174	0.03916	0.0003	0.09142	0.22092	0.27202	0.0058	0.00063	0.01951	1
14	0.13957	0.21664	0.03822	0.00035	0.08764	0.22584	0.26692	0.00658	0.0005	0.01773	1
21	0.13304	0.22492	0.03699	0.00031	0.08086	0.24236	0.26259	0.00546	0.0004	0.01307	1
28	0.13411	0.22983	0.03605	0.00037	0.07708	0.24728	0.25748	0.00624	0.00028	0.0113	1
35	0.14278	0.23136	0.0354	0.00051	0.07632	0.24061	0.25161	0.0089	0.00013	0.01239	1

t	Tr_{71}	Tr_{72}	Tr_{73}	Tr_{74}	Tr_{75}	Tr_{76}	Tr_{77}	Tr_{78}	Tr_{79}	Tr_{711}	SUM_Tr_{7j}
7	0.0712	0.14286	0.0132	0.00069	0.08496	0.05146	0.0558	0.51898	0.0019	0.05895	1
14	0.07418	0.14731	0.01401	0.00087	0.08557	0.0537	0.0572	0.51314	0.00146	0.05256	1
21	0.07181	0.14669	0.01267	0.00112	0.08372	0.06264	0.06086	0.52117	0.00107	0.03824	1
28	0.07479	0.15114	0.01348	0.00131	0.08433	0.06488	0.06226	0.51533	0.00063	0.03185	1
35	0.08313	0.16066	0.01643	0.00142	0.08742	0.06041	0.0614	0.49562	0.00014	0.03337	1

t	Tr_{81}	Tr_{82}	Tr_{83}	Tr_{84}	Tr_{85}	Tr_{86}	Tr_{87}	Tr_{88}	Tr_{89}	Tr_{811}	SUM_Tr_{8j}
7	0.03935	0.06444	0.00543	0.00084	0.03228	0.02152	0.00254	0.76376	0.0054	0.06444	1
14	0.04116	0.06504	0.00504	0.00079	0.0328	0.0218	0.0027	0.75925	0.00409	0.06733	1
21	0.03691	0.06098	0.00512	0.0008	0.03092	0.02244	0.00256	0.77369	0.0029	0.06368	1
28	0.03872	0.06158	0.00473	0.00076	0.03144	0.02272	0.00272	0.76918	0.00159	0.06657	1
35	0.04658	0.06684	0.00386	0.00065	0.03437	0.02263	0.00318	0.74574	0.00015	0.076	1

t	Tr_{90}	Tr_{91}	Tr_{92}	Tr_{93}	Tr_{94}	Tr_{95}	Tr_{96}	Tr_{97}	Tr_{98}	Tr_{99}	Tr_{912}	SUM_Tr_{9j}
7	0.00278	0.00916	0.00188	0.00034	0.00011	0.00021	0.0004	0.00012	0.0075	0.94435	0.03315	1
14	0.00356	0.01143	0.00241	0.00032	0.00022	0.00022	0.00038	0.00019	0.01386	0.92521	0.0422	1
21	0.00359	0.01252	0.00242	0.00032	0.00034	0.00026	0.0002	0.00023	0.02051	0.90932	0.05028	1
28	0.00437	0.01479	0.00294	0.0003	0.00046	0.00027	0.00018	0.0003	0.02688	0.89018	0.05933	1
35	0.0059	0.01824	0.00398	0.00025	0.00057	0.00025	0.00032	0.00038	0.03297	0.86779	0.06935	1

9.2.2 Movers(2)

t	r(t)	Tr_{00}	Tr_{01}	Tr_{02}	Tr_{03}	Tr_{04}	Tr_{05}	Tr_{06}	Tr_{09}	SUM_Tr_{0j}
1	6.25	0.836357	0.147513	0.011266	0.000583	0.00043	0.003543	0.000297	1.17E-05	1
15	6	0.866839	0.119358	0.009804	0.000464	0.000563	0.002676	0.000286	8.91E-06	1
30	7.5	0.910103	0.080038	0.007241	0.00029	0.000595	0.001526	0.000202	3.95E-06	1
45	5.25	0.930873	0.060136	0.006792	0.000215	0.000863	0.000846	0.000272	3.16E-06	1
60	6	0.969638	0.0247	0.004652	6.07E-05	0.000941	-0.00021	0.000219	-9.68E-07	1

t	Tr_{10}	Tr_{11}	Tr_{12}	Tr_{13}	Tr_{14}	Tr_{15}	Tr_{16}	Tr_{19}	SUM_Tr_{1j}
1	0.011458	0.911585	0.063109	0.007338	3.82E-05	0.004589	0.001876	7.36E-06	1
15	0.010545	0.904892	0.071923	0.006593	5.02E-05	0.004303	0.001689	5.94E-06	1
30	0.009578	0.890318	0.088716	0.006588	4.74E-05	0.002985	0.001766	2.22E-06	1
45	0.008587	0.891447	0.089917	0.004903	7.78E-05	0.003813	0.001253	3.17E-06	1
60	0.007615	0.880013	0.103592	0.004561	8.16E-05	0.002924	0.001212	3.87E-07	1

t	Tr_{20}	Tr_{21}	Tr_{22}	Tr_{23}	Tr_{24}	Tr_{25}	Tr_{26}	Tr_{27}	Tr_{28}	Tr_{29}	SUM_Tr_{2j}
1	0.0038	0.279436	0.640676	0.030387	0.001608	0.027048	0.015278	0.001172	0.000536	5.93E-05	1
15	0.003159	0.261759	0.664358	0.027587	0.002267	0.024551	0.01425	0.001278	0.000719	7.26E-05	1
30	0.002153	0.255023	0.676105	0.026985	0.002542	0.020524	0.0145	0.001363	0.000802	3.31E-06	1
45	0.001823	0.222398	0.716757	0.021296	0.003732	0.019363	0.011883	0.001509	0.001127	0.000111	1
60	0.000952	0.210485	0.734285	0.019676	0.00419	0.01591	0.01156	0.001607	0.001258	7.76E-05	1

t	Tr_{30}	Tr_{31}	Tr_{32}	Tr_{33}	Tr_{34}	Tr_{35}	Tr_{36}	Tr_{37}	Tr_{39}	SUM_Tr_{3j}
1	1.83E-07	0.310683	0.280044	0.199704	0.000229	0.05544	0.150874	0.002791	0.000234	1
15	3.26E-06	0.291293	0.305385	0.187101	0.000176	0.057249	0.155684	0.002813	0.000295	1
30	6.60E-06	0.267504	0.34752	0.161833	7.62E-05	0.050726	0.169273	0.003032	2.96E-05	1
45	9.84E-06	0.250108	0.35787	0.161522	6.68E-05	0.062153	0.164968	0.002838	0.000464	1
60	1.32E-05	0.227598	0.393648	0.141244	-1.5E-05	0.059219	0.174978	0.002974	0.000339	1

t	Tr_{40}	Tr_{41}	Tr_{42}	Tr_{43}	Tr_{44}	Tr_{45}	Tr_{46}	Tr_{49}	SUM_Tr_{4j}
1	0.023374	0.111259	0.189933	0.015888	0.638313	0.005062	0.015887	0.000284	1
15	0.020323	0.090687	0.163168	0.011904	0.697069	0.00414	0.012244	0.000464	1
30	0.020275	0.076999	0.141422	0.005876	0.745474	0.003237	0.006449	0.000268	1
45	0.013397	0.045591	0.104974	0.003581	0.824738	0.002154	0.004667	0.000897	1
60	0.011982	0.02836	0.080288	-0.0017	0.879315	0.001215	-0.00033	0.000867	1

t	Tr_{50}	Tr_{51}	Tr_{52}	Tr_{53}	Tr_{54}	Tr_{55}	Tr_{56}	Tr_{59}	SUM_Tr_{5j}
1	0.008659	0.191079	0.095825	0.020856	2.83E-06	0.674897	0.008598	8.35E-05	1
15	0.007371	0.187796	0.113654	0.021995	1.74E-05	0.659585	0.00949	9.05E-05	1
30	0.007341	0.18886	0.154816	0.02643	3.05E-05	0.611037	0.011467	1.92E-05	1
45	0.004449	0.180204	0.149186	0.024047	4.90E-05	0.630671	0.011279	0.000115	1
60	0.003846	0.179325	0.180989	0.027118	6.32E-05	0.595759	0.012822	7.73E-05	1

t	Tr_{60}	Tr_{61}	Tr_{62}	Tr_{63}	Tr_{64}	Tr_{65}	Tr_{66}	Tr_{67}	Tr_{68}	Tr_{69}	SUM_Tr_{6j}
1	8.01E-06	0.170687	0.22772	0.045746	0.00034	0.093101	0.258534	0.200378	0.002742	0.000744	1
15	9.61E-06	0.15247	0.241626	0.042481	0.000319	0.091717	0.244928	0.221089	0.00439	0.00097	1
30	2.25E-11	0.139225	0.258724	0.03587	0.000354	0.083249	0.263269	0.215583	0.003559	0.000166	1
45	1.44E-05	0.112673	0.27116	0.03586	0.000269	0.089598	0.211783	0.268827	0.008236	0.00158	1
60	9.61E-06	0.096767	0.287326	0.03057	0.000279	0.084093	0.216159	0.275071	0.008507	0.001219	1

t	Tr_{70}	Tr_{71}	Tr_{72}	Tr_{73}	Tr_{74}	Tr_{75}	Tr_{76}	Tr_{77}	Tr_{78}	Tr_{79}	SUM_Tr_{7j}
1	2.73E-05	0.11193	0.174786	0.019426	0.000697	0.112072	0.072814	0.07102	0.431519	0.005709	1
15	3.28E-05	0.093804	0.173721	0.016633	0.000791	0.101305	0.070289	0.065389	0.470832	0.007205	1
30	3.91E-09	0.083074	0.182546	0.014314	0.001088	0.10102	0.088181	0.063601	0.465407	0.00077	1
45	4.92E-05	0.053908	0.17023	0.010566	0.000967	0.076867	0.062384	0.052808	0.560837	0.011385	1
60	3.28E-05	0.039491	0.174827	0.007961	0.00118	0.071809	0.071537	0.049219	0.575583	0.00836	1

t	Tr_{80}	Tr_{81}	Tr_{82}	Tr_{83}	Tr_{84}	Tr_{85}	Tr_{86}	Tr_{87}	Tr_{88}	Tr_{89}	SUM_Tr_{8j}
1	3.27E-07	0.054032	0.071243	0.008938	0.000762	0.045851	0.036509	0.004176	0.763105	0.015383	1
15	5.81E-06	0.047231	0.072907	0.007933	0.000847	0.039869	0.030245	0.003636	0.776918	0.020407	1
30	1.18E-05	0.04413	0.077115	0.005534	0.000987	0.037495	0.028591	0.003575	0.798327	0.004235	1
45	1.75E-05	0.03215	0.07618	0.005939	0.001023	0.026562	0.016208	0.002417	0.805716	0.033787	1
60	2.35E-05	0.027273	0.079358	0.004101	0.001142	0.022476	0.012408	0.002136	0.824321	0.02676	1

t	Tr_{90}	Tr_{91}	Tr_{92}	Tr_{93}	Tr_{94}	Tr_{95}	Tr_{96}	Tr_{97}	Tr_{98}	Tr_{99}	SUM_Tr_{9j}
1	0.005693	0.034424	0.003578	0.001819	1.41E-05	0.000696	0.001113	0.000279	0.010605	0.941778	1
15	0.004185	0.024609	0.002602	0.001297	4.71E-05	0.000549	0.000781	0.00022	0.007662	0.958048	1
30	0.002775	0.014576	0.001654	0.000759	8.65E-05	0.000457	0.000426	0.000183	0.004745	0.974339	1
45	0.000929	0.00352	0.000497	0.000176	0.000117	0.000225	7.10E-05	8.98E-05	0.001326	0.993049	1
60	-0.00057	-0.00672	-0.00049	-0.00037	0.000155	0.000106	-0.00028	4.15E-05	-0.00169	1.009824	1

9.2.3 [1-to-3]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	Tr_{GGG}	Tr_{GGI}	Tr_{GGB}	sum_Tr_{GGj}	Tr_{GIG}	Tr_{GII}	Tr_{GIB}	sum_Tr_{GIj}
1	6.25	0.974913	0.024995	0.000093	1.000000	0.386395	0.607523	0.006081	1.000000
15	6	0.974286	0.025525	0.000189	1.000000	0.391323	0.597756	0.010920	1.000000
30	7.5	0.978914	0.020917	0.000169	1.000000	0.447176	0.543127	0.009697	1.000000
45	5.25	0.972302	0.027288	0.000409	1.000000	0.395753	0.582181	0.022066	1.000000
59	6	0.974673	0.024891	0.000436	1.000000	0.429288	0.547432	0.023280	1.000000

t-1	Tr_{GBG}	Tr_{GBI}	Tr_{GBB}	sum_Tr_{GBj}	Tr_{IGG}	Tr_{IGI}	Tr_{IGB}	sum_Tr_{IGj}
1	0.285623	0.156726	0.557651	1.000000	0.970354	0.029516	0.000130	1.000000
15	0.234171	0.118795	0.647034	1.000000	0.971717	0.028013	0.000270	1.000000
30	0.217004	0.050762	0.732233	1.000000	0.990573	0.009178	0.000248	1.000000
45	0.119315	0.040836	0.839849	1.000000	0.972528	0.026881	0.000591	1.000000
59	0.089336	-0.012590	0.923254	1.000000	0.983731	0.015635	0.000634	1.000000

t-1	Tr_{IIG}	Tr_{III}	Tr_{IIB}	sum_Tr_{IIj}	Tr_{IBG}	Tr_{IBI}	Tr_{IBB}	sum_Tr_{IBj}
1	0.118839	0.879729	0.001432	1.000000	0.299296	0.025823	0.674882	1.000000
15	0.091774	0.906820	0.001406	1.000000	0.280939	0.023539	0.695522	1.000000
30	0.039002	0.960302	0.000696	1.000000	0.339098	0.018003	0.642898	1.000000
45	0.036661	0.961908	0.001432	1.000000	0.232171	0.019020	0.748809	1.000000
59	-0.003852	1.002832	0.001020	1.000000	0.257838	0.014989	0.727173	1.000000

t-1	Tr_{BGG}	Tr_{BGI}	Tr_{BGB}	sum_Tr_{BGj}	Tr_{BIG}	Tr_{BII}	Tr_{BIB}	sum_Tr_{BIj}
1	0.960237	0.033121	0.006642	1.000000	0.110047	0.396690	0.493263	1.000000
15	0.966364	0.027296	0.006340	1.000000	0.081249	0.543087	0.375664	1.000000
30	0.971989	0.025607	0.002403	1.000000	0.040025	0.801515	0.158460	1.000000
45	0.979608	0.014261	0.006131	1.000000	0.020798	0.844482	0.134720	1.000000
59	0.985204	0.011011	0.003785	1.000000	-0.013865	1.048335	-0.034469	1.000000

t-1	Tr_{BBG}	Tr_{BBI}	Tr_{BBB}	sum_Tr_{BBj}
1	0.058850	0.021551	0.919599	1.000000
15	0.049369	0.016046	0.934585	1.000000
30	0.035798	0.007193	0.957009	1.000000
45	0.029468	0.004607	0.965925	1.000000
59	0.018057	-0.002570	0.984513	1.000000

9.2.4 [4]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	Tr_{GGG}	Tr_{GGI}	Tr_{GGB}	sum_Tr_{GGj}	Tr_{GIG}	Tr_{GII}	Tr_{GIB}	sum_Tr_{GIj}
1	6.25	0.942558	0.057172	0.000271	1.000000	0.443804	0.549324	0.006872	1.000000
15	6	0.949595	0.050006	0.000399	1.000000	0.481090	0.505977	0.012933	1.000000
30	7.5	0.956779	0.042862	0.000359	1.000000	0.535153	0.454659	0.010187	1.000000
45	5.25	0.964719	0.034585	0.000696	1.000000	0.559279	0.413681	0.027040	1.000000
59	6	0.971555	0.027721	0.000724	1.000000	0.604548	0.367577	0.027875	1.000000

t-1	Tr_{GBG}	Tr_{GBI}	Tr_{GBB}	sum_Tr_{GBj}	Tr_{IGG}	Tr_{IGI}	Tr_{IGB}	sum_Tr_{IGj}
1	0.507212	0.055592	0.437195	1.000000	0.899554	0.099997	0.000449	1.000000
15	0.445657	0.057933	0.496410	1.000000	0.903139	0.096167	0.000693	1.000000
30	0.438656	0.076165	0.485179	1.000000	0.925866	0.073254	0.000879	1.000000
45	0.306609	0.061042	0.632350	1.000000	0.908533	0.090241	0.001226	1.000000
59	0.278400	0.072277	0.649324	1.000000	0.922801	0.075772	0.001427	1.000000

t-1	Tr_{IIG}	Tr_{III}	Tr_{IIB}	sum_Tr_{IIj}	Tr_{IBG}	Tr_{IBI}	Tr_{IBB}	sum_Tr_{IBj}
1	0.219734	0.776140	0.004125	1.000000	0.207878	0.255187	0.536935	1.000000
15	0.205373	0.789485	0.005142	1.000000	0.162072	0.219835	0.618093	1.000000
30	0.174779	0.821808	0.003413	1.000000	0.131678	0.227233	0.641089	1.000000
45	0.176443	0.815896	0.007661	1.000000	0.061652	0.138592	0.799757	1.000000
59	0.153480	0.839437	0.007084	1.000000	0.026414	0.128850	0.844736	1.000000

t-1	Tr_{BGG}	Tr_{BGI}	Tr_{BGB}	sum_Tr_{BGj}	Tr_{BIG}	Tr_{BII}	Tr_{BIB}	sum_Tr_{BIj}
1	0.873985	0.114947	0.011068	1.000000	0.175766	0.745476	0.078758	1.000000
15	0.883216	0.100791	0.015993	1.000000	0.189079	0.721756	0.089165	1.000000
30	0.937305	0.056802	0.005893	1.000000	0.168060	0.819618	0.012322	1.000000
45	0.897641	0.073950	0.028409	1.000000	0.221884	0.655985	0.122131	1.000000
59	0.931873	0.043491	0.024636	1.000000	0.215239	0.701997	0.082764	1.000000

t-1	Tr_{BBG}	Tr_{BBI}	Tr_{BBB}	sum_Tr_{BBj}
1	0.103500	0.070935	0.825565	1.000000
15	0.083016	0.052955	0.864029	1.000000
30	0.067851	0.035513	0.896636	1.000000
45	0.038301	0.014203	0.947496	1.000000
59	0.021653	-0.002746	0.981093	1.000000

9.2.5 [≥5]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	Tr_{GGG}	Tr_{GGI}	Tr_{GGB}	sum_Tr_{GGj}	Tr_{GIG}	Tr_{GII}	Tr_{GIB}	sum_Tr_{GIj}
1	6.25	0.878294	0.120567	0.001138	1.000000	0.440262	0.526250	0.033488	1.000000
15	6	0.884893	0.113426	0.001681	1.000000	0.444171	0.516869	0.038960	1.000000
30	7.5	0.879638	0.118198	0.002164	1.000000	0.458174	0.500480	0.041347	1.000000
45	5.25	0.900526	0.096617	0.002857	1.000000	0.451360	0.497535	0.051106	1.000000
59	6	0.900153	0.096503	0.003344	1.000000	0.460820	0.484568	0.054612	1.000000

t-1	Tr_{GBG}	Tr_{GBI}	Tr_{GBB}	sum_Tr_{GBj}	Tr_{IGG}	Tr_{IGI}	Tr_{IGB}	sum_Tr_{IGj}
1	0.424033	0.217061	0.358906	1.000000	0.742278	0.254530	0.003193	1.000000
15	0.363669	0.179642	0.456689	1.000000	0.748432	0.247867	0.003702	1.000000
30	0.322315	0.170198	0.507487	1.000000	0.759706	0.236491	0.003804	1.000000
45	0.231491	0.095743	0.672766	1.000000	0.761052	0.234102	0.004846	1.000000
59	0.184319	0.075660	0.740021	1.000000	0.769854	0.225042	0.005104	1.000000

t-1	Tr_{IIG}	Tr_{III}	Tr_{IIB}	sum_Tr_{IIj}	Tr_{IBG}	Tr_{IBI}	Tr_{IBB}	sum_Tr_{IBj}
1	0.282510	0.694765	0.022725	1.000000	0.277209	0.192626	0.530165	1.000000
15	0.262341	0.715143	0.022516	1.000000	0.259848	0.179192	0.560960	1.000000
30	0.261214	0.711511	0.027275	1.000000	0.254246	0.192550	0.553204	1.000000
45	0.216638	0.761896	0.021467	1.000000	0.221070	0.147042	0.631888	1.000000
59	0.208055	0.767869	0.024076	1.000000	0.211062	0.149306	0.639633	1.000000

t-1	Tr_{BGG}	Tr_{BGI}	Tr_{BGB}	sum_Tr_{BGj}	Tr_{BIG}	Tr_{BII}	Tr_{BIB}	sum_Tr_{BIj}
1	0.695118	0.284901	0.019982	1.000000	0.219141	0.613262	0.167597	1.000000
15	0.706480	0.267859	0.025662	1.000000	0.228883	0.602486	0.168631	1.000000
30	0.710399	0.256667	0.032933	1.000000	0.201676	0.590197	0.208127	1.000000
45	0.731827	0.230483	0.037689	1.000000	0.254324	0.579484	0.166192	1.000000
59	0.738520	0.217439	0.044040	1.000000	0.242772	0.568287	0.188941	1.000000

t-1	Tr_{BBG}	Tr_{BBI}	Tr_{BBB}	sum_Tr_{BBj}
1	0.166637	0.138237	0.695126	1.000000
15	0.144715	0.112256	0.743029	1.000000
30	0.152036	0.101636	0.746328	1.000000
45	0.094004	0.054494	0.851501	1.000000
59	0.089509	0.038252	0.872238.	1.000000

9.3 Actual Transition Probability P_{ij} (or P_{hij})

9.3.1 Movers(1)

t	r(t)	P_{00}	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{06}	P_{09}	P_{010}	SUM_P_{0j}
7	5.75	0.87371	0.08423	0.00949	0.00037	0.00045	0.00224	0.00035	0	0.02916	1
14	6	0.9054	0.06968	0.00819	0.00044	0.00036	0.00194	0.00022	2.8E-05	0.01375	1
21	7	0.88854	0.0795	0.0081	0.00049	0.00046	0.00176	0.00037	3.1E-05	0.02073	1
28	7.25	0.87629	0.06749	0.00892	0.00011	0.00069	0.0013	0.00033	3.6E-05	0.04484	1
35	6.75	0.90576	0.07041	0.00871	0.00012	0.00081	0.0023	0.00048	0	0.01141	1

t	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{19}	P_{110}	SUM_P_{1j}
7	0.01283	0.90778	0.05802	0.00661	6.3E-05	0.00502	0.00217	7.4E-06	0.00749	1
14	0.01391	0.91107	0.05414	0.00817	5.1E-05	0.00398	0.00165	3.9E-06	0.00702	1
21	0.01192	0.90102	0.06821	0.00548	4.8E-05	0.00363	0.00217	1.3E-05	0.0075	1
28	0.01062	0.87033	0.08332	0.00675	6.7E-05	0.00388	0.00193	9.5E-06	0.0231	1
35	0.01087	0.8547	0.11471	0.00779	9.4E-05	0.00413	0.00196	0	0.00575	1

t	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{210}	SUM_P_{2j}
7	0.00502	0.27079	0.63281	0.02554	0.0038	0.02963	0.01927	0.00267	0.00119	2.8E-05	0.00926	1
14	0.00472	0.27221	0.63818	0.02448	0.00321	0.03019	0.01491	0.0026	0.00177	7.8E-05	0.00765	1
21	0.00319	0.27416	0.64488	0.0241	0.00341	0.02363	0.01641	0.00207	0.00139	4.4E-05	0.00672	1
28	0.00287	0.25379	0.66766	0.02588	0.00315	0.02161	0.01583	0.00231	0.00212	0	0.00479	1
35	0.00215	0.22523	0.69642	0.03138	0.00301	0.01993	0.01514	0.00207	0.00155	6.5E-05	0.00305	1

t	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{39}	P_{310}	SUM_P_{3j}
7	0.28251	0.30045	0.17605	0.00017	0.06361	0.17074	0	0	0.00648	1
14	0.25383	0.2738	0.2612	0.00019	0.05197	0.15067	0	0	0.00834	1
21	0.31189	0.30434	0.16452	0.00017	0.04409	0.16298	0.00498	0.00017	0.00686	1
28	0.25439	0.31618	0.18868	0	0.05311	0.18082	0	0	0.00682	1
35	0.24172	0.33022	0.17701	0	0.05782	0.1759	0.01223	0.00022	0.00489	1

t	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{49}	P_{410}	SUM_P_{4j}
7	0.02356	0.06937	0.15445	0.01178	0.69634	0.00262	0.0144	0	0.02749	1
14	0.0229	0.0458	0.12723	0.0229	0.743	0.00382	0.00891	0	0.02545	1
21	0.01661	0.06168	0.14591	0.00356	0.70937	0.00119	0.01542	0	0.04626	1
28	0.01108	0.06647	0.15811	0.00806	0.72508	0.00201	0.00302	0	0.02618	1
35	0.0127	0.06266	0.11516	0.00085	0.76969	0.00339	0.00593	0	0.02964	1

t	P_{50}	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{59}	P_{510}	SUM_P_{5j}
7	0.00939	0.18422	0.10296	0.01909	7.6E-05	0.66211	0.0136	0.00031	0.00825	1
14	0.00853	0.16186	0.09058	0.0202	0	0.70094	0.00994	8.3E-05	0.00787	1
21	0.00832	0.22674	0.11553	0.02218	0	0.60472	0.01419	0	0.00832	1
28	0.00775	0.16527	0.14369	0.02235	0.00013	0.64026	0.01228	0.00013	0.00814	1
35	0.00716	0.16735	0.18179	0.0381	0	0.58387	0.01381	0	0.00793	1

t	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	P_{610}	SUM_P_{6j}
7	0.13223	0.20744	0.03238	0.00063	0.0752	0.20723	0.32338	0	0.00021	0.02131	1
14	0.14197	0.21308	0.0449	0.00078	0.08591	0.22632	0.26473	0	0	0.02232	1
21	0.15392	0.23139	0.03114	0.00051	0.08506	0.24354	0.24177	0	0.00025	0.01241	1
28	0.13139	0.21702	0.04041	0	0.07466	0.24752	0.27589	0	0.00027	0.01284	1
35	0.10046	0.22107	0.03817	0.00031	0.07603	0.24214	0.31115	0	0.00183	0.00885	1

t	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	P_{711}	SUM_P_{7j}
7	0.054	0.16055	0.01656	0.00072	0.07127	0.07847	0.03744	0.53348	0.00072	0.0468	1
14	0.06102	0.14441	0.01356	0.00068	0.08949	0.04	0.0861	0.49695	0.00136	0.06644	1
21	0.07055	0.14759	0.00864	0.00144	0.08639	0.06263	0.0396	0.54932	0.00072	0.03312	1
28	0.06217	0.15499	0.01226	0	0.07356	0.05079	0.10508	0.50438	0	0.03678	1
35	0.05471	0.14731	0.01178	0.00168	0.08418	0.03872	0.05556	0.58081	0.00421	0.02104	1

t	P_{81}	P_{82}	P_{83}	P_{84}	P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	P_{811}	SUM_P_{8j}
7	0.03862	0.0632	0.00655	0.00023	0.03207	0.01732	0.00398	0.77622	0.00117	0.06063	1
14	0.0316	0.06475	0.00466	0.0013	0.02771	0.01191	0.00104	0.78503	0.00311	0.06889	1
21	0.04143	0.06587	0.00372	0.00027	0.03373	0.0178	0.00266	0.75883	0.00133	0.07437	1
28	0.03835	0.05792	0.00402	0.0008	0.02843	0.02306	0.00268	0.78064	0.00241	0.06168	1
35	0.0279	0.05887	0.00419	0.00056	0.02595	0.01507	0.00279	0.80134	0.01144	0.0519	1

t	P_{90}	P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}	P_{99}	P_{912}	SUM_P_{9j}
7	0.00162	0.00379	0.00108	0.00027	0	0.00054	0.00081	0.00135	0.00189	0.9621	0.02653	1
14	0.00338	0.00677	0.00068	0	0	0	0.00034	0	0.00338	0.92995	0.0555	1
21	0	0.00623	0.00078	0	0	0	0	0	0.00467	0.93069	0.05763	1
28	0	0.00956	0.00119	0	0	0	0	0.00119	0.00358	0.92593	0.05854	1
35	0.00904	0.01506	0	0.00301	0	0	0	0	0.03614	0.68072	0.25602	1

9.3.2 Movers(2)

t	r(t)	P_{00}	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{06}	P_{09}	SUM_P_{0j}
1	6.25	0.82909	0.15478	0.01052	0.00083	0.0007	0.00376	0.00032	0	1
15	6	0.8579	0.12649	0.0107	0.00068	0.00052	0.00344	0.00026	0	1
30	7.5	0.90261	0.08713	0.00684	0.00046	0.00116	0.00166	0.00014	0	1
45	5.25	0.92608	0.06393	0.0072	0.00016	0.00097	0.00133	0.00032	0	1
60	6	0.95058	0.04142	0.00577	0.00013	0.00097	0.00093	0.00021	0	1

t	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{19}	SUM_P_{1j}
1	0.01244	0.9178	0.0573	0.00603	8.7E-05	0.00488	0.00144	1.8E-05	1
15	0.01013	0.88138	0.09424	0.00748	8.8E-06	0.00453	0.00221	4.4E-06	1
30	0.00911	0.90221	0.07907	0.00459	6.8E-05	0.00347	0.00148	4.8E-06	1
45	0.00895	0.90982	0.07271	0.00399	8.1E-05	0.00345	0.001	1E-05	1
60	0.00588	0.88941	0.09667	0.00376	0.0001	0.00315	0.00102	1E-05	1

t	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	SUM_P_{2j}
1	0.00308	0.28662	0.64388	0.02563	0.00161	0.026	0.01175	0.00086	0.00049	8.4E-05	1
15	0.00372	0.24551	0.66932	0.03539	0.00283	0.02278	0.01868	0.00121	0.00053	2E-05	1
30	0.00227	0.27215	0.66459	0.01988	0.00258	0.02333	0.01365	0.00108	0.00047	0	1
45	0.00195	0.24473	0.70064	0.01611	0.00391	0.02007	0.01006	0.00135	0.00098	0.00019	1
60	0.00185	0.19218	0.75119	0.01954	0.00462	0.01667	0.01069	0.00139	0.00104	0.00084	1

t	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{39}	SUM_P_{3j}
1	0	0.2622	0.31971	0.20468	0.0002	0.06446	0.14875	0	0	1
15	0	0.38208	0.25629	0.15942	0	0.04185	0.15354	0.00682	0	1
30	0	0.2953	0.35064	0.14637	0.00021	0.05684	0.14679	0.00385	0	1
45	0	0.24663	0.38636	0.14927	0	0.06538	0.15095	0	0.0014	1
60	0	0.20276	0.40241	0.13759	0.00034	0.07897	0.17069	0.00379	0.00345	1

t	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{49}	SUM_P_{4j}
1	0.01338	0.09365	0.22742	0.02341	0.61538	0.00669	0.02007	0	1
15	0.02392	0.0933	0.1555	0.00478	0.70096	0.00957	0.01196	0	1
30	0.02154	0.09692	0.15231	0.00462	0.70923	0.00154	0.01231	0.00154	1
45	0.01487	0.04936	0.1021	0.00473	0.81812	0.00135	0.00811	0.00135	1
60	0.01285	0.04499	0.09588	0.00482	0.82967	0.00107	0.00696	0.00375	1

t	P_{50}	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{59}	SUM_P_{5j}
1	0.00862	0.19452	0.08916	0.01858	0	0.6804	0.0083	0.00042	1
15	0.00901	0.22223	0.14417	0.02744	0	0.5855	0.01164	0	1
30	0.0081	0.17562	0.14081	0.01748	0	0.64791	0.01009	0	1
45	0.00473	0.19519	0.118	0.01865	0	0.65247	0.01095	0	1
60	0.00437	0.18573	0.16339	0.02385	0.00017	0.60168	0.01881	0.00202	1

t	P_{60}	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	SUM_P_{6j}
1	0	0.15284	0.24521	0.05166	0	0.10483	0.24765	0.19751	0	0.0003	1
15	0	0.16689	0.2228	0.03451	0.00044	0.10179	0.23853	0.21319	0.02184	0	1
30	0	0.14717	0.25798	0.03533	0.00069	0.09228	0.26655	0.19966	0	0.00034	1
45	0	0.11454	0.28114	0.03885	0.0004	0.09211	0.21826	0.2487	0	0.00601	1
60	0	0.10126	0.25745	0.02713	0	0.09729	0.19193	0.27796	0.02912	0.01787	1

t	P_{70}	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	SUM_P_{7j}
1	0	0.09343	0.18394	0.0146	0.00146	0.08321	0.11825	0.04234	0.45985	0.00292	1
15	0	0.12813	0.21588	0.0376	0.00279	0.11142	0.04596	0.05153	0.40669	0	1
30	0	0.10157	0.20073	0.01451	0.00121	0.09311	0.09311	0.04837	0.44619	0.00121	1
45	0	0.05269	0.18323	0.01317	0	0.06707	0.08862	0.1006	0.46587	0.02874	1
60	0	0.06087	0.12174	0.00696	0	0.08348	0.03304	0.0487	0.56348	0.08174	1

t	P_{80}	P_{81}	P_{82}	P_{83}	P_{84}	P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	SUM_P_{8j}
1	0	0.04397	0.06546	0.01172	0.00098	0.03957	0.03078	0.00489	0.78749	0.01514	1
15	0	0.05612	0.08552	0.00962	0	0.04917	0.02726	0.00214	0.76964	0.00053	1
30	0	0.0484	0.09178	0.00776	0.00183	0.04429	0.05571	0.00502	0.74292	0.00228	1
45	0	0.03063	0.06771	0.00443	0.00121	0.02418	0.01169	0.0004	0.77267	0.08706	1
60	0	0.03182	0.06155	0.00261	0	0.02087	0.01095	0.00522	0.76056	0.10642	1

t	P_{90}	P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}	P_{99}	SUM_P_{9j}
1	0.00388	0.02907	0	0	0	0	0.00194	0	0.01163	0.95349	1
15	0	0.02421	0	0	0	0	0	0	0	0.97579	1
30	0.00791	0.04348	0	0	0	0	0.00395	0.00395	0.00791	0.93281	1
45	0.00073	0.0022	0.00073	0	0	0	0	0	0	0.99634	1
60	0	0.00198	0.00088	0	0	0	0.00044	0	0.00044	0.99627	1

9.3.3 [1-to-3]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	P_{GGG}	P_{GGI}	P_{GGB}	sum_P_{GGj}	P_{GIG}	P_{GII}	P_{GIB}	sum_P_{GIj}
1	6.25	0.96996	0.0299	0.00014	1	0.34994	0.64405	0.00601	1
15	6	0.97908	0.02077	0.00015	1	0.42199	0.57248	0.00553	1
30	7.5	0.97279	0.02703	0.00018	1	0.41813	0.56664	0.01523	1
45	5.25	0.97909	0.02023	0.00068	1	0.42746	0.54856	0.02398	1
59	6	0.97954	0.01975	0.00071	1	0.44023	0.53009	0.02968	1

t-1	P_{GBG}	P_{GBI}	P_{GBB}	sum_P_{GBj}	P_{IGG}	P_{IGI}	P_{IGB}	sum_P_{IGj}
1	0.28571	0.2381	0.47619	1	0.93827	0.06173	0	1
15	0.33333	0.16667	0.5	1	0.98492	0.01508	0	1
30	0.2	0	0.8	1	0.99028	0.00972	0	1
45	0.03846	0.03846	0.92308	1	0.93602	0.06398	0	1
59	0.17647	0	0.82353	1	0.87902	0.11675	0.00423	1

t-1	P_{IIG}	P_{III}	P_{IIB}	sum_P_{IIj}	P_{IBG}	P_{IBI}	P_{IBB}	sum_P_{IBj}
1	0.16543	0.83081	0.00376	1	0.17333	0.09333	0.73333	1
15	0.07627	0.92316	0.00057	1	0.40741	0	0.59259	1
30	0.05076	0.94825	0.00099	1	0.61111	0	0.38889	1
45	0.02248	0.97586	0.00166	1	0.20779	0.02597	0.76623	1
59	0.04018	0.95817	0.00165	1	0.16049	0.18519	0.65432	1

t-1	P_{BGG}	P_{BGI}	P_{BGB}	sum_P_{BGj}	P_{BIG}	P_{BII}	P_{BIB}	sum_P_{BIj}
1	0.91837	0.06122	0.02041	1	0.32143	0.60714	0.07143	1
15	0.90698	0.09302	0	1	0.25	0.75	0	1
30	0.92105	0.05263	0.02632	1	0.14286	0.71429	0.14286	1
45	1	0	0	1	0	1	0	1
59	0.66667	0.25	0.08333	1	0.28571	0.57143	0.14286	1

t-1	P_{BBG}	P_{BBI}	P_{BBB}	sum_P_{BBj}
1	0.07135	0.01784	0.91081	1
15	0.06416	0.02817	0.90767	1
30	0.06744	0.01349	0.91908	1
45	0.03125	0.01042	0.95833	1
59	0.0161	0.00997	0.97393	1

9.3.4 [4]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	P_{GGG}	P_{GGI}	P_{GGB}	sum_P_{GGj}	P_{GIG}	P_{GII}	P_{GIB}	sum_P_{GIj}
1	6.25	0.94375	0.05591	0.00034	1	0.42456	0.5634	0.01204	1
15	6	0.95522	0.04472	5.4E-05	1	0.49375	0.49554	0.01071	1
30	7.5	0.94554	0.05389	0.00057	1	0.50058	0.48899	0.01043	1
45	5.25	0.97338	0.0263	0.00031	1	0.57474	0.3866	0.03866	1
59	6	0.97137	0.02788	0.00075	1	0.46853	0.48077	0.0507	1

t-1	P_{GBG}	P_{GBI}	P_{GBB}	sum_P_{GBj}	P_{IGG}	P_{IGI}	P_{IGB}	sum_P_{IGj}
1	0.33333	0.16667	0.5	1	0.84385	0.15615	0	1
15	0.375	0	0.625	1	0.93212	0.06788	0	1
30	0.25	0	0.75	1	0.94218	0.05675	0.00107	1
45	0.27778	0.11111	0.61111	1	0.79576	0.20313	0.00112	1
59	0.27273	0.04545	0.68182	1	0.86174	0.13636	0.00189	1

t-1	P_{IIG}	P_{III}	P_{IIB}	sum_P_{IIj}	P_{IBG}	P_{IBI}	P_{IBB}	sum_P_{IBj}
1	0.12858	0.85672	0.0147	1	0.35	0.025	0.625	1
15	0.20642	0.79083	0.00274	1	0.17647	0.11765	0.70588	1
30	0.18961	0.80617	0.00422	1	0.25	0.375	0.375	1
45	0.11651	0.87385	0.00964	1	0.09524	0.09524	0.80952	1
59	0.13275	0.85615	0.0111	1	0.19444	0.08333	0.72222	1

t-1	P_{BGG}	P_{BGI}	P_{BGB}	sum_P_{BGj}	P_{BIG}	P_{BII}	P_{BIB}	sum_P_{BIj}
1	0.875	0.125	0	1	0.16667	0.7	0.13333	1
15	0.83333	0.16667	0	1	0.25	0.75	0	1
30	0.94737	0.05263	0	1	0.3	0.7	0	1
45	0.83333	0.125	0.04167	1	0.15789	0.57895	0.26316	1
59	0.88	0.12	0	1	0.1	0.9	0	1

t-1	P_{BBG}	P_{BBI}	P_{BBB}	sum_P_{BBj}
1	0.07639	0.09722	0.82639	1
15	0.08197	0.08197	0.83607	1
30	0.05051	0.0303	0.91919	1
45	0.02353	0.01176	0.96471	1
59	0.02837	0.01064	0.96099	1

9.3.5 [≥5]-Mover

G = Good I = Indeterminate B = Bad

t-1	av_r	P_{GGG}	P_{GGI}	P_{GGB}	sum_P_{GGj}	P_{GIG}	P_{GII}	P_{GIB}	sum_P_{GIj}
1	6.25	0.89593	0.10271	0.00136	1	0.4098	0.55642	0.03378	1
15	6	0.88831	0.11004	0.00165	1	0.44877	0.51027	0.04096	1
30	7.5	0.87238	0.12536	0.00226	1	0.45698	0.49747	0.04555	1
45	5.25	0.92653	0.07076	0.00271	1	0.43987	0.50686	0.05327	1
59	6	0.93808	0.05975	0.00217	1	0.45203	0.50174	0.04622	1

t-1	P_{GBG}	P_{GBI}	P_{GBB}	sum_P_{GBj}	P_{IGG}	P_{IGI}	P_{IGB}	sum_P_{IGj}
1	0.29032	0.29032	0.41935	1	0.7557	0.24141	0.00289	1
15	0.24324	0.24324	0.51351	1	0.78045	0.2169	0.00265	1
30	0.35714	0.13265	0.5102	1	0.78349	0.21279	0.00372	1
45	0.21918	0.10959	0.67123	1	0.71396	0.2806	0.00544	1
59	0.2233	0.01942	0.75728	1	0.75894	0.23276	0.0083	1

t-1	P_{IIG}	P_{III}	P_{IIB}	sum_P_{IIj}	P_{IBG}	P_{IBI}	P_{IBB}	sum_P_{IBj}
1	0.24592	0.73051	0.02357	1	0.2537	0.17778	0.56852	1
15	0.2741	0.70458	0.02132	1	0.25813	0.18356	0.55832	1
30	0.30535	0.67208	0.02257	1	0.29577	0.22711	0.47711	1
45	0.13298	0.84381	0.02322	1	0.18893	0.10115	0.70992	1
59	0.15927	0.82303	0.0177	1	0.15584	0.12662	0.71753	1

t-1	P_{BGG}	P_{BGI}	P_{BGB}	sum_P_{BGj}	P_{BIG}	P_{BII}	P_{BIB}	sum_P_{BIj}
1	0.69524	0.27619	0.02857	1	0.23485	0.60227	0.16288	1
15	0.7462	0.23427	0.01952	1	0.23675	0.63958	0.12367	1
30	0.78669	0.19178	0.02153	1	0.23919	0.63359	0.12723	1
45	0.70801	0.2584	0.03359	1	0.25561	0.56502	0.17937	1
59	0.72222	0.21296	0.06481	1	0.21698	0.61321	0.16981	1

t-1	P_{BBG}	P_{BBI}	P_{BBB}	sum_P_{BBj}
1	0.11922	0.09333	0.78745	1
15	0.17247	0.12874	0.69879	1
30	0.19826	0.10516	0.69658	1
45	0.07979	0.04258	0.87764	1
59	0.03066	0.0129	0.95644	1